DEM-FEM model of the outflow of backfilling sand from behind a seawall under wave motion

Lechoslaw G. Bierawski¹, Shiro Maeno², Hitoshi Gotoh³ and Eiji Harada⁴

The DEM and FEM techniques are combined to numerically model the collapse of a structure consisting of a fixed vertical revetment and backfilling sand under wave impacts in a vertical domain. The course of events inside the granular material in the vicinity of the wall during its washing out is the object of this research. The study is based on the methods of computational mechanics of sediment transport; hence the sediment and pore water oscillation interaction and the inter-particle collisions are considered. Small-scale laboratory tests were performed to provide verification of the numerical model.

Introduction

Many cases of coastal structures destruction by waves have been reported. Unfortunately, the discussion on the explanation of the ruination, by H. Ouemeraci (1994), indicates that the present design approaches cannot explain most of the failure modes. Trying to contribute to development of new construction methods we decided to investigate the outflow of backfill from behind a seawall caused by pore water oscillations. The investigated structure could be a seawall, a bulkhead or a revetment, which is narrow and vertical. Its main purpose is to retain or prevent sliding of the land, with the secondary purpose of affording protection to the upland property from damage by wave action (Shore Protection Manual, 1977). Unfortunately, it turns out that even a local scour in the seabed in front of such a structure rapidly increases the risk that the outflow would occur (see Photo 1). This paper treats the mechanism of the collapse of backfilling sand under these conditions. The work is based on studies of many researchers, e.g. Yamamoto (1981), Nago (1982), Zen (1984), and it is continuation of preceding research made by Nago and Maeno (1987 and 1993), Nago et al., (1995) Maeno et al. (1999 and 2002). The dynamic behavior of the sandy bed in the vicinity of a seawall and the possibility for the outflow of the backfilling sand was clarified there. The present study is focused on the motion of particular grains around the seawall during the collapsing process, which is still not clarified. To investigate the phenomenon a numerical model has been developed and small-scale experiments conducted. In the proposed numerical model, the FEM (Finite Elements Method) and the DEM (Distinct Elements Method) are coupled. Obtained sequential pore water pressure distributions and related moves of grains provide fine data for discussion of the phenomenon. Additionally, the outflow preventing effect of higher seawall foundations is considered by using the model.

Outline of the experiment

The experimental apparatus, designed for the small-scale laboratory tests is presented in Fig.1. Its dimensions are: 80 cm in length, 70 cm in height, and 29.5 cm in width. A board of 1 cm thick was fixed as the seawall. The backfill as well as the sandy bed was made of the

¹ Doctoral Course Student, Graduated School of Natural Science and Technology, Okayama University, Okayama, Japan, lbier@cc.okayama-u.ac.jp
² Dr. of Eng., Assoc. Professor, Dept. of Environmental and Civil Eng., Okayama University, 700-8530 Tsushima-Naka, Okayama, Japan, ph. 0862518151, fax 0862518257, maeno@cc.okayama-u.ac.jp
³ Dr. of Eng., Assoc. Professor, Department of Civil Engineering, Kyoto University
⁴ Dr. of Eng., JSPS Post-Doctoral Research Fellow, Department of Civil Engineering, Kyoto University
highly saturated Toyoura sand. Its parameters are: the median diameter $d_{50} = 0.25$ mm, the specific gravity $G_S = 2.65$. The sand was placed into the container and vibrated to obtain the specific homogenous characteristics: the porosity $n \cong 0.4$, and the coefficient of permeability $k = 1.2 \times 10^{-2}$ cm/s. The height of the sandy bed was 40 cm. Additionally, in order to visualize the movement of sand around the wall several layers of colored sand were introduced. A table type pressure load was applied to the bed surface. In the paper a regular wave of around 40 cm high and 1 Hz frequency is considered. Water pressure was recorded at Pt.1, and pore water pressure was measured at several points numbered from 2 to 7. Their arrangement is shown in Fig. 1. Moreover, photography was used to save sequential configurations of the backfilling sand and sandy bed in a beforehand-assumed time interval. The top surface of the backfilling sand was open to atmospheric pressure. The experiment was done for two distinct cases: the first one for the wall foundations of 5 cm in height (Case 1), and the second one of 10 cm (Case 2). Each test lasted 2,000 minutes.

Outline of the numerical model

The numerical model was developed in a two dimensional vertical domain. The Distinct Element Method is used for tracking the movements of particles around the seawall during the outflow. The Finite Element Method was applied to model the wave-induced pore water oscillations in the sand bed. These two numerical techniques were coupled so that the FEM mesh was constantly updated according to sequential rearrangements of the particles. In return, the calculated forces acting on the sand particles were corrected by pore water pressure values coming from the FEM computations.

DEM summary

P.A.Cundall (1971) initiated the Distinct Element Method in the framework of the studies on models for rock-mass behavior in the 70s. The method is based on use of an explicit numerical scheme in which the interaction of the particles is monitored contact by contact and the motion modeled particle by particle. Realistic friction laws and simple stiffness parameters govern it. The DEM modification the used in the present study was done by Gotoh and Sakai (1997) as an application for computational mechanics of sediment transport. In this research, the sand particles are modeled as rigid balls placed between two vertical walls, which keep the same distance as the diameter of the particle. The friction between the two confining walls and the balls equals zero. Among each ball, the spring and dashpot
systems are introduced to express the particle interaction (see Fig.2). Equations of motion of the \(i\)-th particle in the vertically two dimensional domain are as follows:

\[
\rho_w \left\{ \frac{\rho_s}{\rho_w} + C_M \right\} \frac{\pi}{6} d^3 \frac{du_{pi}}{dt} = \sum_j \left\{ -f_n \cos \alpha_{ij} + f_s \sin \alpha_{ij} \right\}_j + F_{Dx} \\
\rho_w \left\{ \frac{\rho_s}{\rho_w} + C_M \right\} \frac{\pi}{6} d^3 \frac{dw_{pi}}{dt} = \sum_j \left\{ -f_n \sin \alpha_{ij} - f_s \cos \alpha_{ij} \right\}_j - \rho_w \left( \frac{\rho_s}{\rho_w} - 1 \right) \frac{\pi}{6} d^3 g + F_{Dz} \\
\rho_s \frac{\pi d^5}{60} \frac{d\omega_{pi}}{dt} = \frac{d}{2} \sum_j \{ f_s \}_j
\]

Where: \(t\) is the time, \(\rho_w\) is the density of water, \(\rho_s\) is the density of sediment particles, \(C_M\) is the added mass coefficient, \(d\) is the diameter of sediment particles, \(u_{pi}, w_{pi}\) are the velocities of the \(i\)-th particle in horizontal \(x\) and vertical \(z\) direction, respectively, \(\alpha_{ij}\) is the contacting angle between the \(i\)-th and \(j\)-th particles, \(g\) is the gravity acceleration, \(\omega_{pi}\) is the angular velocity of the \(i\)-th particle, \(f_n, f_s\) are the the normal (\(n\)) and tangential (\(s\)) components of the force acting on the contacting plane between the \(i\)-th and \(j\)-th particles, \(F_{Dx}, F_{Dz}\) are the seepage forces in the horizontal and vertical direction. In this simulation model following values of parameters were taken according to Gotoh et al. (2000): \(C_M = 0.5\), \(\Delta t = 1.0 \times 10^{-4}\) (s), \(\rho_w = 1.0\) (g/cm\(^3\)), \(\rho_s = 2.65\) (g/cm\(^3\)). The assumed drag coefficient was \(\mu = 0.176\).

**FEM summary**

The Finite Elements Method is used to model propagation of pore water oscillations, induced by wave impacts, inside the sand bed. Compressible pore water and a small amount of gas bubbles inside the bed were considered independently. To obey Darcy’s law in the model the poro-elastic equations 4, 5 and 6 were applied (Biot, 1941, Verruijt, 1969, Nago and Maeno, 1984):

\[
G \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial z^2} \right) + \frac{G}{1-2\nu} \frac{\partial}{\partial x} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) = \rho g \frac{\partial h}{\partial x}
\]
Where: $\lambda_w$ is the water-holding porosity, $\lambda_a$ is the aeration porosity, $k$ is the permeability coefficient, $G$ is the shear modulus, $\beta$ is the compressibility of water, $\nu$ is the Poisson’s ratio, $g$ is the gravity acceleration, $u_x, u_z$ are the incremental displacements in $x$ and $z$ directions respectively, $h$ is the incremental pore water pressure.

In the computation were used values as follows:

$$G = 3.5 \times 10^7 \text{(N/m}^2\text{)}, \quad \beta = 4.3 \times 10^{11} \text{(m}^2\text{/kN)} \quad k = 1.2 \times 10^{-2} \text{(cm/s)} \quad \lambda_w = 0.003 \quad \lambda_a = 0.40 \quad \nu = 0.45$$

The boundary conditions at the sandy bed line correspond to regular waves. There the normal stress $\sigma$ equals zero, and the water pressure $p$ varies due to the wavy motion. At the top surface of the backfill the effective normal stress $\sigma$ is also equal to zero but the pressure is atmospheric. For the other still boundaries we assumed no existence of pore flow $\frac{\partial h}{\partial n} = 0$.

### Analytical conditions

As mentioned before, two cases were investigated: 5 cm seawall foundations in height (Case 1), and 10 cm (Case 2). This distinction was made in order to provide some data to discuss the influence of the foundations height on the structure stability. In the numerical model these two cases were also considered. Moreover the calculations were run for two particle diameters $d_1 = 1.0 \text{ cm}$, and $d_2 = 0.5 \text{ cm}$. Because of the computer memory limitation and time consumption, in the case of the smaller diameter (Case 1c and Case 2c) the DEM domain was a part of the FEM domain only, as shown in Photo 2. Parameters of all the calculated cases are compared in Table 1.

### Coupling routine

The coupling of the above-described DEM and FEM numerical methods was crucial for the investigation. The FEM mesh was updated every $\Delta t = 0.01 \text{ s}$ (every 100 cycles of DEM) according to the change of the bed surface particle positions. After the mesh adjustment, new pressure values at each nodal point were interpolated based on the previous mesh data. In return, the calculated forces acting on the sand particles were corrected by pore water pressure values obtained from the FEM computations. The auto-generated mesh let us rebuild it...
according to the current positions of the particles located at the bed line (see Fig.3). The shape of the mesh has significant influence on pore water pressure propagation in the sand bed. The pore water pressure gradients were taken into account during the calculation of the forces acting on the particles. The seepage force in Eq.1 and Eq.2 can be expressed by equations as follows:

$$F_{Dx} = \frac{\rho_w g \pi d^4}{6} \times b_x, \quad F_{Dz} = \frac{\rho_w g \pi d^4}{6} \times b_z$$

(7)

Here $b_x$ and $b_z$ are the values of the hydraulic gradients in the horizontal and vertical direction respectively. The seepage force was calculated at the time step of $\Delta t = 0.01$ s and interpolated at the time step set for the DEM part.

**Results and discussion**

**Experimental results**

Photo 3 and 4 show the sand bed and the backfill configuration in the vicinity of the seawall at the foundations 5cm in height (Case 1) and 10cm in height (Case 2) after 2000 minutes of wave loading. The influence of foundations height on perseverance of outflow is illuminated very well here. While the outflow was significant in Case 1, it was only slight in Case 2. Fig.4 presents the large difference in the amounts of the washed out sand in these cases. However comparing Photo 3 with Photo 4 it is noticeable that at the end of the experiment the range of moved sand is quite similar. The load from the backfill causes pushing out the sand located in front of the wall. The seepage force, generated within the backfill due to the pore water pressure gradient distribution favors the phenomenon of outflow stirring the particles within a thin layer under the seawall. Namely, it is significant by the wall and decays with distance. The sand in front of the wall is lifted and replaced by the backfill. Such process is
Fig. 6 Seepage force distribution

a) $t = 0.25s$ (under wave crest)  
b) $t = 0.75s$ (under wave trough)  
a) $t = 5s$ (the end of calculations)

Fig. 7 Grain arrangement of Case 1a

a) $t = 0.25s$ (under wave crest)  
b) $t = 0.75s$ (under wave trough)  
a) $t = 5s$ (the end of calculations)

Fig. 8 Grain arrangement of Case 1b

a) $t = 0.25s$ (under wave crest)  
b) $t = 0.75s$ (under wave trough)  
a) $t = 5s$ (the end of calculations)

Fig. 9 Grain arrangement of Case 1c

a) $t = 0.25s$ (under wave crest)  
b) $t = 0.75s$ (under wave trough)  
a) $t = 5s$ (the end of calculations)
repeated by each wave and oriented towards equilibrium state between the loads from the backfilling sand and the sand in front of the wall. The outflow becomes less intensive with its progress. It is visible in Fig.4 (the change of the inclination of the graph line) and in Fig.5, which shows that most of the sand flowed out within the first 500 minutes. The sand accumulated in front of the wall together with the lowered load from the remaining backfill hinder the outflow. Because of the harmonic characteristics of the phenomenon, the peak of the outflow form locates in a certain distance from the wall, depending on the seepage force rate.

**Numerical results and their comparison to the test results**

Fig.6 presents distribution of the pore pressure gradients (the seepage force) under wave crest ($t = 0.25$ s), wave trough ($0.75$ s) and these obtained at the end of the calculations ($5.0$ s). In Fig.7 to Fig.9 the corresponding grain arrangements are shown. During a trough of wave the seepage force is directed seaward (Fig. 6b), whereas under a wave crest it has the opposite course (Fig.6a). The seepage force affects the sand particles so that the wave trough conditions initiate the outflow of the backfill, and wave crests cause the sand grains to be pushed back. However they cannot reach their former positions, because the sand below the wall is partially replaced by the higher located particles of the backfill. Additionally the outflow is damped by the particles located in front of the wall. At the beginning of the collapse the prevention of the outflow is not significant. In this situation the load from the backfilling sand still located behind the wall additionally pushes out the lower placed grains. The numerical model illustrates this relation very well. The outflow movement is concentrated in a narrow zone by the seawall. It is well modeled in Case 1c only, where the grain diameter is smaller and equals 0.5 cm. In the other cases the layer of movement is much thicker so that almost all the sand bed is affected. Only in Fig.9 (Case 1c) one can notice that
the low layers of colored sand remains straight as in Photo 3. The shift of the outflow form peak is pronounced in Fig.7c, Fig.8c, and Fig.9c. Moreover the distance corresponds to the experiment. The mode of backfill collapsing is slightly different from observed in the experiment. However the model can be found good enough to explain the outflow phenomenon. The application of the uniform diameter of grains in the numerical approach causes its imperfections. Identical round particles under changes of stress conditions rearrange to obtain the hexagonal layout, and it leads to difficulty with dilating by shearing. Fig.10 and Fig.11 present the wall with the foundations of 10 cm in height. In all the cases of high foundations the final calculation state (after 5.0 s) is almost the same as at the initial state (after 0.1 s). Although the seepage force generates some movement, any outflow does not occur. It provides a brief illustration of the significant influence of the foundations height on structure safety.

**Conclusions**

This research was based on the assumption that the backfilling sand can be destroyed by flowing out from behind a seawall under cyclic pore water pressure changes. This phenomenon was examined and clarified here by using the experiment and the numerical DEM-FEM model. From these results we obtained one can conclude as follows. Any changes in structure foundations depth conditions are very dangerous for the structure stability, because even a local scour in front of a seawall can lead to the final collapse by causing the outflow of the backfilling sand under wavy action. The outflow of the backfilling sand is slow and at beginning sometimes unnoticeable. It is irreversible and always leads to serious damage or at least devastates the top surface at the backfill. The intensity of the outflow depends mainly on several factors: the height of the wall, the backfill and seabed characteristics, the wall foundations height as well as the acting wave parameters. In this paper we also proved the good applicability of the DEM-FEM numerical model for reproducing the sequence of events during destruction of grain material based constructions. Despite of some imperfections this manner is found to be possible to improve and apply successfully to various tasks.

**References**


