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Frequency Dependence Deterioration of AC Resistance in Large-Diameter Litz Wire for High Power Induction Heating

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Abstract— This paper reports the frequency dependence deterioration of the ac resistance found in Litz wires for high power applications. Litz wire is widely utilized for heating coils of the induction heaters as a wire with small ac resistance at high frequency. Particularly, the recent spread of next-generation switching devices resulted in high-frequency operation of high-power induction heaters, which increasingly adopt Litz wire with a large diameter for copper loss reduction. However, little knowledge has been reported on the effect of the Litz wire diameter on the frequency dependence of the ac resistance, which will be important for the design of Litz wire for high power applications. To elucidate the effect, this paper experimentally measured and compared the frequency dependence of the ac resistance among the Litz wires with different diameters. The result revealed that the Litz wire with a large diameter exhibits worse frequency dependence of the ac resistance than that with a small diameter. In other words, the Litz wire with a large diameter increased ratio of the ac resistance to the dc resistance at a lower frequency than that with a small diameter. An analysis of this reason was also carried out using the recently proposed Litz wire copper loss model, which revealed that the copper loss by the internal proximity effect tends to prevail in the large-diameter Litz wire to cause deterioration of the frequency dependence of the ac resistance. The analysis also suggested that sparse distribution of the strands as well as thinner strands can be effective to mitigate this effect, which can be a key to solve this problem in the Litz wire for high power applications.

Keywords—Ac resistance, copper loss, induction heating, Litz wire, proximity effect

I. INTRODUCTION

The Litz wire is widely utilized for the heating coils of the induction heaters [1]–[4]. The Litz wire is made by twisting many thin strands in multiple levels hierarchically [5] as illustrated in Fig. 1. Owing to this hierarchical twisting structure, all the strands equally experience every part of the cross-section, which ideally enables the homogeneous current distribution in the Litz wire cross-section. Because the Litz wire can thus mitigate the concentrated current distribution due to the skin effect [6] and the proximity effect [7][8], the Litz wire exhibits small ac resistance at a high frequency.

Conventionally, the Litz wire has been commonly utilized for low-power induction heaters, where the ac current of the heating coil is below 100A approximately. The reason is that high-power induction heaters have been commonly designed to operate at a low frequency, in which the skin and proximity effects scarcely take place. However, driven by the recent spread of the next-generation switching devices such as SiC-MOSFETs and GaN-HEMTs, the high-power induction heaters are becoming to be operated at higher frequencies. Consequently, the heating coils of high-power induction heaters increasingly adopt the Litz wire to reduce the copper loss.

These Litz wires for high-power induction heaters tend to carry a large ac current and therefore should have a larger diameter than those for low-power induction heaters. However, these large-diameter Litz wires are recently becoming applied in the industry and currently have a smaller market size compared to the normal Litz wires with smaller diameters. Therefore, little knowledge has been known on the optimal design of these large-diameter Litz wires. Among the various missing knowledge of the large-diameter Litz wire design, the effect of the diameter on the frequency dependence of the ac resistance is particularly important, because the excessive copper loss of the Litz wire can disastrously damages the heating coil in high power induction heating.

The purpose of this paper is to elucidate the effect of the Litz wire diameter on the frequency dependence of the ac resistance. This paper firstly conducted an experiment that evaluates and compares the ac resistance of Litz wires with various diameters, which are all made of the same strands. The

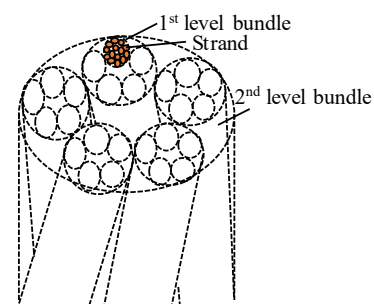


Fig. 1. Typical Litz wire structure with 3 levels of twisting.

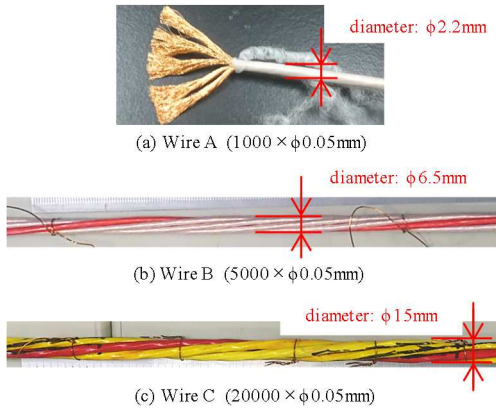


Fig. 2. Photographs of experimental Litz wires.

TABLE I. SPECIFICATIONS OF EXPERIMENTAL LITZ WIRES

Parameter	Wire A	Wire B	Wire C
Radius of strand α_s [mm]		0.025	
Radius of Litz wire α_L [mm]	1.1	3.25	7.5
Number of strands in 1st level bundle n_s	40	40	40
Total number of strands in Litz wire n_{total}	1000	5000	20000
Twisting level	3	4	5
Packing factor η	0.52	0.3	0.22

experiment results revealed that the Litz wire with a larger diameter exhibits worse frequency dependence of the ac resistance than that with a smaller diameter. In other words, the Litz wire with a larger diameter increased the ratio of the ac resistance to the dc resistance at a lower frequency than that with a smaller diameter. To elucidate the mechanism of this frequency dependence deterioration, this paper further theoretically analyzed the copper loss in a large-diameter Litz wire. This analysis also suggested the possible design to mitigate this frequency dependence deterioration.

This analysis is performed based on the Litz wire copper loss model recently proposed in [9]. Among a number of preceding Litz wire loss models [10]–[19], this model was chosen because it can predict the frequency dependence of the ac resistance by only the physical constants and the structural parameters of the Litz wires, which can be all determined from the Litz wire construction.

The subsequent discussions comprise 3 sections. Section II presents the experiment that compares the ac resistance among Litz wires with different diameters. Then, section III presents the copper loss analysis after giving a brief review of the copper loss model utilized in this analysis. Finally, section V gives the conclusions.

II. EXPERIMENT

A. Experiment Set-up

An experiment was carried out to elucidate the effect of the diameter on the ac resistance of the Litz wire. For this purpose, this experiment constructed the Litz wires with three different diameters made of the same strands and compared the frequency dependence of the ac resistance among these Litz wires.

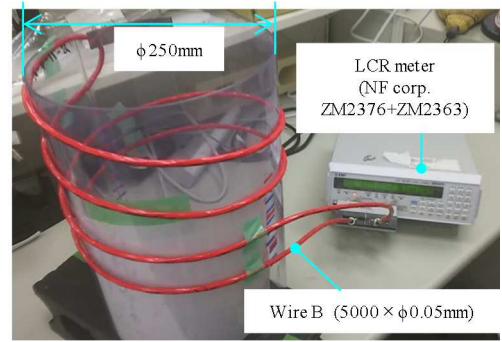


Fig. 3. Non-inductive solenoid for ac resistance measurement.

Figure 2 shows the photographs of these Litz wires. These Litz wires were all made to have the same length of 3m. Wire A, i.e. the thinnest wire with a diameter of 2.2mm, is a commercially available Litz wire supplied by Elekrisola Dr. Gerd Schildbach GmbH & Co. KG, made by twisting 1,000 strands with the diameter of 50 μ m in 3 levels. Wire B, i.e. the Litz wire with the diameter of 6.5 mm, is made by twisting 5 wires A at the twisting pitch of approx. 150mm, and therefore it contains 5,000 strands. Wire C, i.e. the thickest Litz wire with the diameter of 15mm, is made by further twisting 4 wires B at the twisting pitch of approx. 270mm, and therefore it contains 20,000 strands. Therefore, these experimental Litz wires were all made of the same strands. The detailed specifications of these Litz wires are summarized in Table I.

These Litz wires were wound to form non-inductive solenoids, which incorporate the two counterclockwise turns and the two clockwise turns implemented alternately on a plastic tube with a diameter of 250mm as shown in Fig. 3. Then, these experimental solenoids are connected to the LCR meter (NF Corp. ZM2376 and ZM2363) to measure the ac resistance in the frequency range from 1kHz to 1MHz. This non-inductive solenoid coil was chosen to test the ac resistance under the condition without the proximity effect caused by the neighboring turns.

Certainly, the heating coil of the practical induction heating system is generally designed to generate the intense magnetic field in the coil by winding the turns in the same direction, i.e. clockwise or counterclockwise. Therefore, each turn of the heating coil is applied with the ac magnetic field generated by the neighboring turns except for the ac magnetic field generated by local ac current in this winding turn. However, as the ac voltage applied to the heating coil tends to be extremely large in high-power applications, the turns of the heating coil are commonly placed not adjacently but separately with sufficient vacant space between the neighboring turns for galvanic isolation. Furthermore, as the recent induction heating system tends to be operated at a high frequency greater than hundreds of kHz, the winding turns should also be placed separately to reduce the parasitic capacitance of the coil for separating the operating frequency from the self-resonance frequency of the heating coil. Consequently, the proximity effect caused by the neighboring turns tends to have less effect on the copper loss than the internal proximity effect, which is the proximity effect caused by the local ac current flow inside the Litz wire. Figure 4 depicts the mechanism of the internal proximity effect, where the eddy current is generated inside the strands of the Litz wire due to the local ac current flowing through the Litz wire itself.

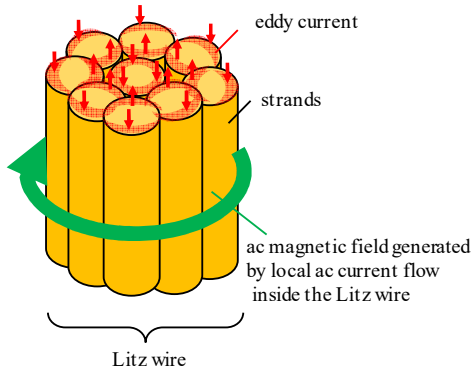


Fig. 4. Conceptual diagram of internal proximity effect, in which eddy current is induced by ac magnetic field generated by local ac current flow in Litz wire.

The experimental non-inductive solenoids can be therefore expected to have a similar electromagnetic condition at the wire as the heating coil of a high-power induction heating system, which results in a similar copper loss as the heating coil. Neglecting the proximity effect by the neighboring turns is particularly convenient for this study investigating the effect of the Litz wire diameter on the ac resistance because the copper loss is not affected by the heating coil geometry but only by the Litz wire structure under this assumption, as explained in the next section.

B. Results

Figure 5 shows the measurement results of the solenoid coils of the three Litz wires. Figure 5(a) shows the ac resistance; Fig. 5(b) shows the ratio of the ac resistance to the dc resistance. As can be seen in Fig. 5(a), the thickest wire, i.e. wire C, exhibited the smallest ac resistance among the three Litz wire throughout the whole frequency region of the measurement, as is expected from the largest cross-section area of the wire. However, among these three wires, wire C exhibited the greatest increase rate of the ac resistance by the frequency increase. This feature was prominently observed in the ratio of ac resistance to dc resistance. Figure 5(b) showed that the ratio increased at lowest frequency in wire C among these three wires. This indicates that wire C exhibits the worst frequency dependence of the ac resistance, although the experimental Litz wires are made of the same strands.

The ac resistance measurement results of wire C shows the small gap at the frequency of 200kHz. The reason for this gap appears to be originated from the measurement instrument. The LCR meter switched its measurement range around 200kHz, which may have caused this small gap in the ac resistance measurement results. This gap can occur also in the measurement results of wires A and B. However, these wires have far greater ac resistance than wire C, which have made this gap appear subtle in the measurement results. On the other hand, wire C was made with an extremely large diameter and therefore exhibited small ac resistance at high frequencies as 200kHz, which may have caused the gap to appear non-negligibly in the measurement results.

Consequently, the experiment suggested that the frequency dependence of the ac resistance was deteriorated as the Litz wire diameter increases, which can be an obstacle for applying the Litz wire for high power applications.

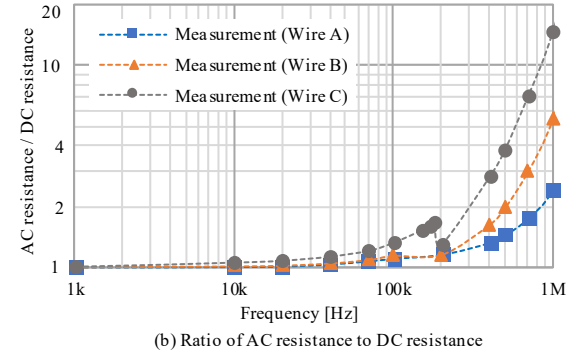
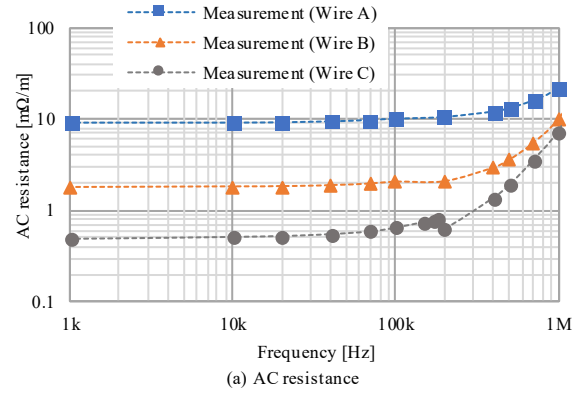


Fig. 5. Measurement results of ac resistance and ratio between ac and dc resistances of three experimental Litz wire.

III. ANALYSIS OF FREQUENCY DEPENDENCE DETERIORATION IN THICK LITZ WIRE

This section analyses the reason for the frequency dependence deterioration of the ac resistance found in the experiment of the large-diameter Litz wire. For this purpose, this section analytically estimates the breakdown of the ac resistance into contribution of different copper loss mechanisms. This estimation is based on the recently proposed copper loss model of the Litz wire [9]. Hereafter, this section firstly gives a brief review of this copper loss model before discussing this estimation.

A. Brief Review of Copper Loss Model

The copper loss of a wire can be generally expressed as the sum of the two types of losses: One is the copper loss generated by the current flow through the wire, and the other is the copper loss generated by the ac magnetic field from outside the part of wire under consideration of copper loss. The former and latter copper losses are proportional to the square of the root-mean-square value of the current and the ac magnetic field, respectively. In many practical coils, the ac magnetic field is applied perpendicular to the wire. Therefore, a basic form of the copper loss per unit length of a wire can be expressed as

$$p_c = R_L I_{rms}^2 + G_L H_{ac}^2, \quad (1)$$

where p_c is the copper loss per unit length, I_{rms} is the root-mean-square value of the current flowing through the wire, and H_{ac} is the root-mean-square of the intensity of the ac magnetic field applied to the wire. Coefficients R_L and G_L are the parameters that characterize the former and latter copper loss respectively; both of these coefficients dependent on the wire structure.

The copper loss model [9] of the Litz wire determines the formulae to calculate these coefficients depending on the structural parameters of the Litz wire. According to [9], these coefficients are given as

$$R_L = \frac{m\rho}{\pi\alpha_s^2 n_{total}} F(\gamma_s) F(\gamma_b) + \frac{4\pi\rho n_{total} K(\gamma_s)}{8\pi^2 \alpha_L^2} \left(\frac{4m^3}{3} - \frac{13m}{6} + \frac{11}{6m} \right), \quad (2)$$

$$G_L = 4\pi\rho n_{total} K(\gamma_s) \left(\frac{3m}{4} + \frac{1}{4m} \right). \quad (3)$$

where ρ is the resistivity of the copper; α_s is the radius of the strand; n_{total} is the total number of the strands in the Litz wire; α_L is the radius of the Litz wire; m , γ_s , and γ_b are the parameter defined as (4) and (6), respectively; F and K are functions of real number x defined as (9) and (10), respectively.

Parameter m is the ratio of the strand length l_s and the Litz wire length l_L . Hence, m is defined as

$$m = \frac{l_s}{l_L}. \quad (4)$$

The strand length l_s is commonly difficult to be measured by mechanical means. Instead, the strand length can be determined by measuring the dc resistance of the Litz wire. If the dc resistance of the Litz wire denotes R_{dc} , l_s can be determined as

$$l_s = \frac{n_{total} \pi \alpha_s^2}{\rho} R_{dc}. \quad (5)$$

Parameter γ_s and γ_b represent the degree of the skin effect in the strand and the 1st level bundle, which is the bundle corresponding to the lowest level of twisting as illustrated in Fig. 1. The Litz wire is made of multiple levels of twisting. The 1st level bundle is the bundles made by directly twisting the strands. The 2nd level of twisting further twist multiple 1st level bundles; by further twisting the 2nd level bundles, the Litz wire can be constructed. Parameter γ_s and γ_b are proportional to the ratio of the radius of the strand or the 1st level bundle to the skin depth taking place in the level of the strand or the 1st level bundle. Therefore,

$$\gamma_s = \alpha_s \sqrt{\frac{\omega \mu_0}{\rho}}, \quad \gamma_b = \alpha_b \sqrt{\frac{\omega \mu_0 \eta}{\rho F(\gamma_s)}}, \quad (6)$$

where ω is the angular frequency, μ_0 is the permeability of the air, α_b is the radius of the 1st level bundle, and η is the packing factor of the 1st level bundle. Parameters α_b and η are also difficult to be directly measured. However, [9] proposes to estimate these two parameters by the following equations made of the structural parameters of the Litz wire:

$$\alpha_b = \sqrt{\frac{n_s}{n_{total}}} \alpha_L. \quad (7)$$

$$\eta = \frac{n_{total} \alpha_s^2}{\alpha_L^2}. \quad (8)$$

Functions F and K represent the effects of the skin effect and the proximity effect, respectively, on the copper loss. These functions are defined as

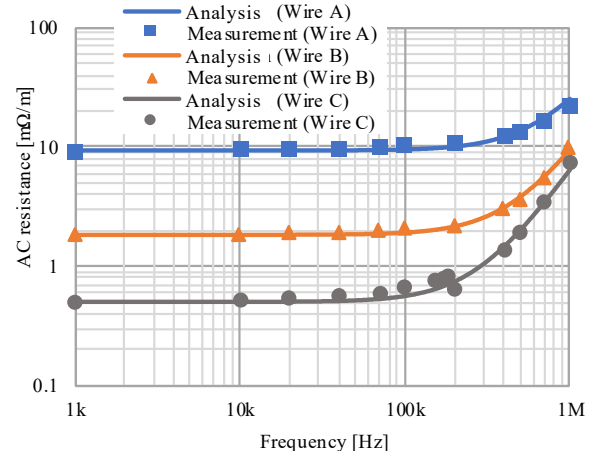


Fig. 6. Comparison of theoretically predicted ac resistance per unit length to the measurement results of experimental non-inductive solenoids.

$$F(x) \equiv \frac{x \operatorname{ber} x \operatorname{bei}' x - \operatorname{ber}' x \operatorname{bei} x}{2 (\operatorname{ber}' x)^2 + (\operatorname{bei}' x)^2}, \quad (9)$$

$$K(x) \equiv -x \frac{\operatorname{ber}_2 x \operatorname{ber}' x + \operatorname{bei}_2 x \operatorname{bei}' x}{(\operatorname{ber} x)^2 + (\operatorname{bei} x)^2}, \quad (10)$$

where ber and bei are Kelvin functions, which are the special functions related to the Bessel function [20].

According to (1), the copper loss per unit length is determined by the current I_{rms} and the ac magnetic field H_{ac} applied to the wire. However, in many practical coils for power electronics, H_{ac} is caused by the neighboring turns of the same wire. Therefore, the copper loss is only dependent on the current I_{rms} because H_{ac} is proportional to I_{rms} . Noting that the proportionality coefficient $k(x)$ between H_{ac} and I_{rms} is dependent on the position x along the wire of the coil, (1) can be rewritten as

$$p_c = \{R_L + G_L k^2(x)\} I_{rms}^2. \quad (11)$$

Therefore, the total copper loss P_c of the Litz wire coil can be obtained by integrating (11) along the wire:

$$P_c = \int_S p_c ds = \left\{ \int_S (R_L + G_L k^2) ds \right\} I_{rms}^2 = \left(R_L l_L + \int_S G_L k^2 ds \right) I_{rms}^2, \quad (12)$$

where S is the curve of the Litz wire and ds is the line element.

The ac resistance R_{ac} of a wire is the proportional coefficient between the total copper loss P_c and the square of the current I_{rms}^2 . Therefore, R_{ac} can be expressed as

$$R_{ac} = R_L l_L + \int_S G_L k^2 ds. \quad (13)$$

B. Confirmation of Copper Loss Model

This subsection confirms that this copper loss model, shown in the previous subsection, is consistent with the experimental results. For this purpose, the theoretical ac resistance values was calculated for the experimental Litz wires, and the results were compared with the experimental measurement results reported in the previous section.

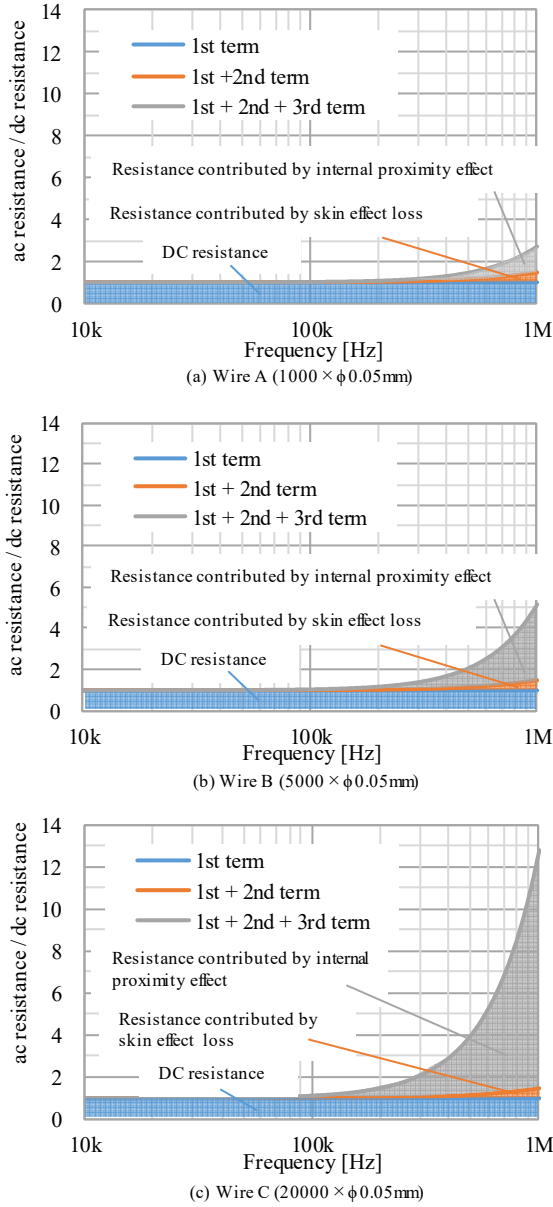


Fig. 7. Breakdown of ratio of ac resistance to dc resistance into three copper loss mechanisms.

The experimental results were observed in the Litz wire coil wound as the non-inductive solenoid. This solenoid has clockwise turns and counterclockwise turns alternatingly. Therefore, the proximity effect caused by the neighboring turns has a small contribution to the ac resistance of the coil because the magnetic field generated by the neighboring turns can cancel each other. Consequently, the ac resistance of the experimental non-inductive solenoid coils can be approximated simply as follows by neglecting the second term of (13).

$$R_{ac} \approx R_L I_L. \quad (14)$$

Under this assumption, the analytical calculation only needs the parameters of the Litz wire construction. Therefore, the analytical R_{ac} was calculated based on parameters listed in Table I and compared to the experimental measurement results. Figure 6 shows the results. As can be seen in the figure, the analytical calculation agrees well with the experimental

results, supporting the appropriateness of the analysis based on the copper loss model.

C. Breakdown of Copper Loss Mechanisms

According to (2), R_L is determined only by the Litz wire construction and unaffected by the heating coil geometry. Therefore, (14) suggests that the reason for the frequency dependence deterioration of the ac resistance, found in the experiment, lies in the Litz wire structure.

As discussed in this section, the induction heating coil for high-power applications tends to have a distance between the neighboring turns, and therefore the proximity effect caused by the neighboring turns can also have smaller contribution to the ac resistance of the coil. This implies that the high-power induction heating system can also need to improve the Litz wire structure rather than the coil geometry in order to avoid this frequency dependence deterioration of the ac resistance.

For elucidating the reason for the deterioration, R_L was broken down into the contributions of the three different copper loss mechanisms and analytically investigated which mechanism resulted in this deterioration.

According to (2), the right-hand side is made of two terms. The first term contains function $F(x)$. This term represents two copper loss mechanisms: One is the copper loss caused by the dc resistance; the other is the copper loss caused by the skin effect in the strands as well as the bundles in the Litz wire. On the other hand, the second term contains function $K(x)$. This term represents the internal proximity effect, which is the copper loss caused by the proximity effect due to the local ac current flow inside the Litz wire.

For distinguishing these three mechanisms, (2) is rewritten in the following form:

$$R_L = \frac{m\rho}{\pi\alpha_s^2 n_{total}} + \frac{m\rho}{\pi\alpha_s^2 n_{total}} \{F(\gamma_s)F(\gamma_b) - 1\} + \frac{4\pi\rho n_{total} K(\gamma_s)}{8\pi^2 \alpha_L^2} \left(\frac{4m^3}{3} - \frac{13m}{6} + \frac{11}{6m} \right). \quad (15)$$

In (15), each term of the right-hand side represents one of the three different copper loss mechanisms: The first term is the dc resistance, the second term is the skin effect, and the third term is the internal proximity effect.

Based on this equation, the contribution of these three terms was investigated in the analytically calculated ac resistance per unit length of the experimental non-inductive solenoids. Figure 7 shows the breakdown of the copper loss mechanism of the three Litz wires. This figure plots the ac resistance normalized by the dc resistance to compare the contribution of three copper loss mechanisms among the three Litz wires.

As can be seen in this figure, the contribution of the dc resistance and the skin effect was the same among the three Litz wires, whereas the internal proximity effect cause the difference among the Litz wires. This indicates that the internal proximity effect has caused the frequency dependence deterioration of the ac resistance in the large-diameter Litz wire.

D. Discussions

The reason for the prominent internal proximity effect in the large-diameter Litz wire lies in the third right-hand term

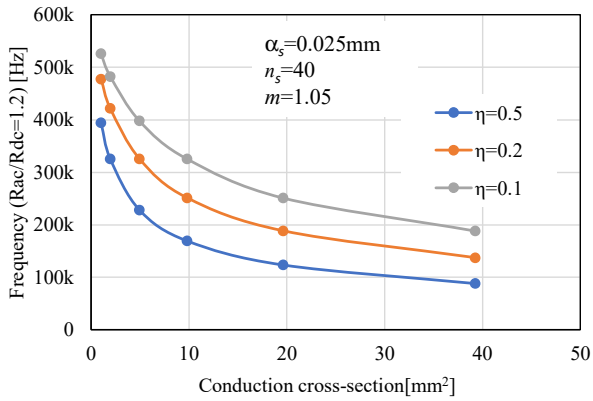


Fig. 8. Analytical calculation results of the lowest frequency where ratio between ac and dc resistance increases above 1.2 at various packing factor. Strand diameter and number of strand in 1st level bundle was set constant.

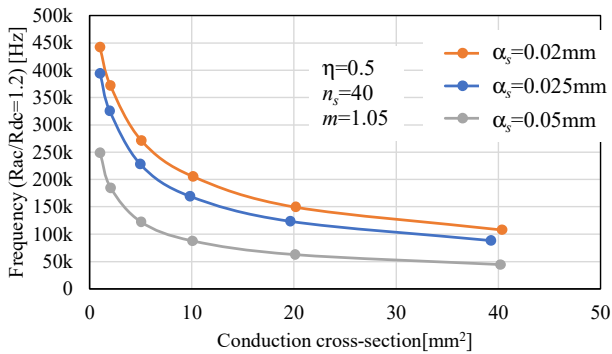


Fig. 9. Analytical calculation results of the lowest frequency where ratio between ac and dc resistance increases above 1.2 at various strand radius. Packing factor and number of strand in 1st level bundle was set constant.

of (15). The ac resistance contributed by the internal proximity effect loss, which is represented by the third right-hand term, is proportional to n_{total}/α_L^2 , considering that $K(\gamma_s)$ and m are scarcely affected by the Litz wire radius α_L . If the number of the strands in a unit area of cross-section is approximated to be constant regardless of the Litz wire diameter, n_{total} is proportional to α_L^2 , and therefore the third term is almost constant regardless of the Litz wire radius α_L .

Meanwhile, the dc resistance and the resistance contributed by the skin effect loss, which are represented by the first and second right-hand terms, respectively, become smaller as α_L increases. Therefore, the ratio of the internal proximity effect loss to the dc resistance increases prominently as the Litz wire radius α_L increases. This resulted in the increase in the ratio of the ac resistance to the dc resistance in the Litz wire with a large diameter, causing the frequency dependence deterioration of the ac resistance.

As can be seen from the discussion, an effective method to mitigate the internal proximity effect in the large-diameter Litz wire is to reduce the ratio n_{total}/α_L^2 , or the packing factor η . The effectiveness of reducing the packing factor is also supported by the copper loss model. Figure 8 shows the analytical calculation result when the packing factor is varied without changing the strand radius and the number of strands inside a 1st level bundle. This figure plots the lowest frequency where the ratio of the ac resistance to the dc resistance increases above 1.2. The horizontal axis is the conduction cross-section, which is the net copper area in the Litz wire

cross-section. As can be seen in the figure, a smaller packing factor can improve the frequency dependence of the ac resistance, although a small packing factor will lead to an increase in the Litz wire diameter.

Another well-known method to mitigate the proximity effect is to adopt thinner strands. Figure 9 shows the analytical calculation result when the strand radius is varied without changing the packing factor and the number of strands inside a 1st level bundle. This figure also plots the lowest frequency where the ratio of the ac resistance to the dc resistance increases above 1.2. The horizontal axis is the conduction cross-section. The result exhibits that a smaller strand radius can improve the frequency dependence of the ac resistance, although an extremely small strand radius will need enormous number to form the Litz wire and cause difficulty in manufacturing.

To summarize, reduction of the packing factor and reduction of the strand radius are effective remedies to mitigate the internal proximity effects, although these remedies entail severe drawbacks in practical applications. Therefore, the analysis results of this section suggest that the Litz wire design of high-power induction heaters needs further improvement of the technology to suppress the internal proximity effect.

IV. CONCLUSIONS

This paper investigated the effect of the diameter on the ac resistance of the Litz wire. As a result, the Litz wire with a large diameter was found to exhibit a worse frequency dependence of the ac resistance compared to the Litz wire with a small diameter, even if the same strands are twisted with the same packing factor to construct the Litz wire. The analysis based on the recently proposed Litz wire copper loss model revealed that this problem is caused by the internal proximity effect, which prevails in the ac resistance of the large-diameter Litz wire.

Analysis also revealed straightforward remedies to mitigate this problem, i.e. reduction of the packing factor and reduction of the strand diameter. However, these remedies also have the drawbacks of increase in the Litz wire diameter or manufacturing difficulty. Therefore, the analysis results of this paper suggest the importance to improve the technology to suppress the internal proximity effect for Litz wires of high-power induction heaters.

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