# Application of extremum co-energy principle for homogenizing current distribution in parallel-connected windings in transformers: Design optimization of winding turn allocation among winding layers

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# Application of Extremum Co-Energy Principle for Homogenizing Current Distribution in Parallel-Connected Windings in Transformers: Design Optimization of Winding Turn Allocation among Winding Layers

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## **Keywords**

«Transformer», «Passive Component», «Design», «Ohmic Losses»

#### Abstract

Parallel-connected windings are widely utilized for power transformers to reduce the copper loss. However, this reduction effect is often hindered by the proximity effect, which causes concentrated current distribution in parallel-connected windings. This paper addresses this issue by proposing a design optimization method of winding turn allocation among winding layers. This optimization method is based on the recently proposed insight on the proximity effect, referred to as the extremum co-energy principle. Experiment on a planar transformer with four primary winding layers was carried out to verify the proposed method. As a result, the proposed method successfully optimized the allocation of the winding turns to minimize the AC resistance.

#### Introduction

Parallel-connected windings are commonly utilized for high frequency power transformers of isolated DC-DC converters. The DC-DC converters are generally required to improve the efficiency. As a result, the power transformers for these converters are often intensely required for reducing the power loss. As widely known, one of the major causes of the power loss in the power transformer is the copper loss. The copper loss is the ohmic loss caused by the parasitic resistance of the primary and secondary windings. In order to reduce the copper loss, parallel-connected windings are effective because they can conveniently increase the wire cross-sectional area for the primary or secondary current [1]–[7].

As naturally supposed, the parallel-connected windings can effectively reduce the copper loss generated by the DC current. Because the DC current distribution is dependent on the parasitic resistance of the winding, the DC current tends to be distributed uniformly among the parallel-connected winding of the same cross-sectional area. However, the AC current is not necessarily distributed uniformly [1]–[7]. In high frequency operation, the proximity effect is a dominant factor that affects the AC current distribution [1], [4], [6]–[9]. This effect can cause concentration of the AC current into one winding of parallel-connected windings, thus deteriorating the reduction effect of the copper loss. As pointed out in literature [8], [10], [11], the proximity effect is deeply affected by the physical transformer structure such as disposition of the windings. Therefore, elucidation of the transformer structure that can homogenize the AC current distribution in parallel-connected windings is intensely required to achieve effective copper loss reduction for high frequency transformers.

For this purpose, there have been proposed a number of analytical and numerical methods that can predict the AC current distribution in parallel-connected windings based on the transformer structure [3], [4], [6], [7], [10], [11]. For example, [4], [10], [11] utilized numerical methods based on FEM-based electromagnetic analysis to predict the AC current distribution of a magnetic device. Although these methods are proven to be effective for accurate prediction of the AC current distribution, these methods may often suffer from complicated analysis procedure or complicated model construction, if applied for



Fig. 1: Generalized transformer model with parallel-connected windings.

practical transformer design. On the other hand, [3], [6], [7] proposed analytical methods. These methods are beneficial in simple model construction. However, they still tend to have comparatively complicated calculation process, which may require partial utilization of numerical calculation of magnetic field or complicated equivalent circuit. As we have seen, many of the preceding methods need complicated analysis or modeling procedure, which may cause difficulties in solving the inverse problem, i.e. derivation of a physical transformer structure from requirement of homogenized AC current distribution.

To cast a new light on this issue, a novel analytical method has been proposed in [12][13]. This method is based on a novel insight referred to as the extremum co-energy principle, which states that the AC current is distributed so that the total magnetic co-energy [14][15] of the transformer takes the extremum. As exemplified in the previous studies [12][13], this method is proven to be beneficial in straightforward analysis with simple calculation for predicting the AC current distribution. Hence, this method may be effective for deriving a transformer structure that homogenizes the AC current distribution in parallel-connected windings.

The purpose of this paper is to show the effectiveness of the extremum co-energy principle for analyzing and deriving the transformer structure to homogenize the AC current distribution. For this purpose, this paper presents a practical example based on a planar transformer with multiple winding layers with parallel-connected windings. The extremum co-energy principle was utilized to optimize the winding turn allocation among the winding layers to homogenize the AC current distribution.

The following discussion consists of 4 sections. The next section presents a brief review of the extremum co-energy principle. Then, section 3 presents optimization of the winding turn allocation based on this principle. Section 4 presents the experimental verification. Finally, section 5 presents the conclusions.

### **Extremum Co-Energy Principle**

According to the extremum co-energy principle, the AC current in the parallel-connected transformer windings are distributed so that the total magnetic co-energy of this transformer takes the extremum under the condition in which the total primary current and the total secondary current are given. Detailed discussion of the reason is discussed in the preceding studies [12][13]. However, we present a concise review of the reason below, based on a generalized model of a transformer with parallel-connected windings.

Figure 1 illustrates the generalized transformer model. This magnetic component has n windings. Windings 1, 2, ..., k are the primary windings connected in parallel, whereas windings k+1, k+2, ..., n are the secondary windings connected in parallel. For convenience, we assume that no DC current nor DC voltage is applied to the windings, as is natural for transformers. In addition, the extremum coenergy principle imposes an assumption that the frequency of the AC current is high enough so that the AC current distribution is affected by the magnetic coupling between the windings, i.e. the proximity effect and scarcely affected by the resistivity of the conductor of the windings. Hence, under this assumption, the parasitic resistance of the conductor is neglected. Let  $i_j$  and  $v_j$  be the AC current and the AC voltage of winding j, respectively. In addition, let  $\psi_j(i_1, i_2, ...)$  be the magnetic flux linkage of winding j expressed as a function of winding current  $i_1, i_2, ...$  If we introduce the vector  $\mathbf{i}$  and  $\mathbf{\psi}$  defined as  $\mathbf{i} \equiv [i_1, i_2, ..., i_n]^t$  and  $\mathbf{\psi} \equiv [\psi_1, \psi_2, ..., \psi_n]^t$ , the total magnetic co-energy of this transformer model can be formulated as

$$E_{co}(\mathbf{i}) = \int_{0}^{1} \boldsymbol{\psi}(\mathbf{i}) \cdot d\mathbf{i}, \tag{1}$$

where  $E_{co}$  is the magnetic co-energy as a function of current vector **i**, and **0** is the zero vector.

Now, we consider an arbitrary infinitesimal virtual change  $\delta i_1, \delta i_2, ..., \delta i_k$  in the AC current of windings 1, 2, ..., k without changing the total primary current. Hence, we impose  $\delta i_1 + \delta i_2 + ... + \delta i_k = 0$ . If we denote the change in the magnetic co-energy as  $\delta E_{co}$ ,  $\delta E_{co}$  can be expressed as

$$\delta E_{co} = \Psi_1 \delta i_1 + \Psi_2 \delta i_2 + \dots + \Psi_k \delta i_k.$$
<sup>(2)</sup>

According to Faraday's law, we have the following relation between  $\psi_i$  and  $v_i$ .

$$\Psi_j = \int v_j dt. \tag{3}$$

Noting that the voltage of the parallel-connected windings must be the same, we have  $v_1=v_2=...=v_k$  and  $v_{k+1}=v_{k+2}=...=v_n$ . Because there is no DC voltage applied to the winding, we obtain the following relation for the magnetic flux linkage:

$$\Psi_1 = \Psi_2 = \dots = \Psi_k, \quad \Psi_{k+1} = \Psi_{k+2} = \dots = \Psi_n.$$
 (4)

Substituting (4) into (2), we have

$$\delta E_{co} = \Psi_1 \left( \delta i_1 + \delta i_2 + \dots + \delta i_k \right) = 0.$$
<sup>(5)</sup>

The result indicates that the current in the parallel-connected primary windings must be distributed so that the magnetic co-energy takes the extremum. Similarly, the current in the parallel-connected secondary windings must also be distributed so that the magnetic co-energy takes the extremum, according to the similar discussion.

#### Homogenization of Current Distribution in Parallel-Connected Windings

The extremum co-energy principle can be utilized to derive a transformer structure that homogenizes the AC current distribution in the parallel-connected windings. In this section, we discuss homogenization of the current distribution based on a simple example, shown in Fig. 2.

Figure 2 shows the sectional view of a planar transformer with EI core. The transformer has two parallel-connected primary windings and one secondary winding. Each primary windings forms two winding layers: Winding 1 forms layers A and D; and winding 2 forms layers B and C. For convenience,



Fig. 2: Analyzed planar transformer structure with two parallel-connected primary windings and one secondary winding

we assume that the windings are wound uniformly to form a winding layer and all the winding layers are disposed at equal distances. We also assume that the primary windings are made of the twisted thin wire strands and the secondary winding is made of a solid conductor. In addition, we assume that the permeability is the core material is far larger than that of the air, as is natural for high frequency power transformers with the ferrite core.

Let  $N_A$ ,  $N_B$ ,  $N_C$ , and  $N_D$  be the number of turns assigned to winding layers A–D, respectively. We impose  $N_A+N_D=N_B+N_C$  so that primary windings 1 and 2 have the same number of turns to avoid the main flux passing through the center leg of the core from violating Faraday's law at the parallelconnected primary windings. The AC current in windings 1 and 2 are denoted as  $i_1$  and  $i_2$ , respectively. Then, we solve the current distribution between windings 1 and 2 using the extremum co-energy principle.

As discussed in [12], the magnetic co-energy of this transformer is mainly contributed by the coenergy stored in the space occupied by the windings and the space between the winding layers, if the permeability of the magnetic core is sufficiently large. However, the skin effect deeply affects the current distribution in the cross-section of the winding. As a result, the magnetic field is generally weak inside the solid conductor or inside the winding made of the twisted thin wire strands [16][17], resulting in small co-energy contributed by the space occupied by the windings. Therefore, for simple calculation, the space occupied by the windings are neglected for the calculation of the total magnetic co-energy. In other words, the total magnetic co-energy is formulated by summing the co-energy stored in the space between the winding layers.

The magnetic field may have local inhomogeneity even in the space between the winding layers, particularly if the winding conductor does not entirely fulfil the winding layer. Therefore, accurate calculation of the co-energy stored in this space is generally complicated. However, for simple calculation, we employ another well-known approximation, referred to as the Dowell approximation [18]–[25], that magnetic field is uniform between the winding layers.

According to the Dowell approximation, magnetic field  $H_{AB}$  in the space between layers B and C can be obtained as (6) by applying Ampere's law along the dashed line in Fig. 2.

$$H_{BC} = \frac{N_A \dot{i}_1 + N_B \dot{i}_2}{w},\tag{6}$$

where w is the width of the window space. Similarly, we can obtain magnetic field  $H_{AB}$  in the space between layers A and B, field  $H_{CD}$  in the space between layers C and D, and field  $H_{DS}$  in the space between layer D and the secondary winding as

$$H_{AB} = \frac{N_A i_1}{w}, \quad H_{CD} = \frac{N_A i_1 + N_B i_2 + N_C i_2}{w}, \quad H_{DS} = \frac{N_A i_1 + N_B i_2 + N_C i_2 + N_D i_1}{w}.$$
 (7)

The magnetic co-energy per unit volume can be expressed as  $\mu H^2/2$  in linear media, where  $\mu$  is the permeability and *H* is the magnetic field. Hence, the total magnetic co-energy  $E_{co}$  can be formulated as

$$E_{co}(i_{1},i_{2}) = \frac{1}{2}\mu_{0}H_{AB}^{2}Sh + \frac{1}{2}\mu_{0}H_{BC}^{2}Sh + \frac{1}{2}\mu_{0}H_{CD}^{2}Sh + \frac{1}{2}\mu_{0}H_{DS}^{2}Sh$$
$$= \frac{\mu_{0}Sh}{2w^{2}} \Big\{ N_{A}^{2}i_{1}^{2} + (N_{A}i_{1} + N_{B}i_{2})^{2} + (N_{A}i_{1} + N_{B}i_{2} + N_{C}i_{2})^{2} + (N_{A}i_{1} + N_{B}i_{2} + N_{C}i_{2})^{2} + (N_{A}i_{1} + N_{B}i_{2} + N_{C}i_{2})^{2} \Big\}$$
(8)

where  $\mu_0$  is the permeability of the air, and *S* and *h* are the top surface area and the height of the space between the winding layers.

Now, we impose that the total primary current  $i_{total} = i_1 + i_2$  is given. Substituting  $i_2 = i_{total} - i_1$  into (8) and utilizing the relation  $N_A + N_D = N_B + N_C$ , we have

$$E_{co} = \frac{\mu_0 Sh}{2w^2} \Big[ N_A^2 i_1^2 + \{ N_B i_{total} + (N_A - N_B) i_1 \}^2 + \{ (N_B + N_C) i_{total} + (N_A - N_B - N_C) i_1 \}^2 + \{ (N_B + N_C) i_{total} \}^2 \Big]$$
(9)

According to the magnetic co-energy principle,  $i_1$  can be determined so that  $E_{co}$  takes the extremum. Hence, we can determine  $i_1$  by substituting (9) into  $\partial E_{co}/\partial i_1=0$ . If we denote the ratio between  $i_1$  and  $i_{total}$  as  $\kappa$ , we have the following relation:

$$\kappa = \frac{i_1}{i_{total}} = -\frac{N_B (N_A - N_B) + (N_B + N_C) (N_A - N_B - N_C)}{N_A^2 + (N_A - N_B)^2 + (N_A - N_B - N_C)^2}.$$
(10)

The ratio  $\kappa$  predicted by (10) can take a positive or negative value. The meaning of the negative value is that the AC current in primary winding 1 flows in the opposite direction to the total primary current. Hence, the AC current of negative  $i_1$  has a phase angle 180 degrees different from that of the total primary current.

As a simple example, we consider the common design of  $N_A$ ,  $N_B$ ,  $N_C$ , and  $N_D$ , in which  $N_A$ ,  $N_B$ ,  $N_C$ , and  $N_D$  are assigned with the same number of turns. Then, we obtain  $i_1=i_{total}$  and  $i_2=0$  according to (10). Therefore, the current will be concentrated in winding 1 in this common design. This indicates that the parallel-connected windings have entirely no effect to reduce the copper loss. Therefore, this design is not beneficial and should be improved.

Next, we seek for the solution of the homogeneous current distribution, i.e.  $\kappa = 1/2$  or  $i_1 = i_2 = i_{total}/2$ . For example, we assume  $N_B = N_C$ . We denote  $N_A/N_B$  as  $\alpha$ . (Hence,  $N_D/N_B = 2-\alpha$ .) Then, according to (10),  $\kappa$  can be rewritten as

$$\kappa = \frac{i_1}{i_{total}} = \frac{5 - 3\alpha}{3\alpha^2 - 6\alpha + 5}.$$
(11)

Finally, the solution of  $\kappa = 1/2$  is obtained as  $\alpha = \sqrt{5/3} \approx 1.29$ . This result indicates the optimum winding turn allocation among winding layers:  $N_A$  and  $N_D$  should be designed as  $N_A = 1.28N_B$  and  $N_D = 0.71N_B$  to homogenize the AC current among primary windings 1 and 2.



Fig. 4: Photographs of windings for experimental transformer

E core		TDK PC40EER49-Z
I core		TDK PC40EI50-Z
Secondary winding		Copper plate (thickness: 1mm)
Inner and outer diameter of secondary winding	$r_a, r_b$	9.0mm, 18mm
Primary windings		Twisted wire strands
		2UEWSTC 57/φ0.1
Number of turns (winding layer B and C)	$N_B, N_C$	7T
Number of turns (winding layer A)	$N_A$	5~10T
Number of turns (winding layer D)	$N_D$	9~4T
Height of the space between the winding layers	h	1.0mm
Height of the winding layer		1.0mm

**Table 1: Specifications of experimental transformer** 

As we have seen above, the extremum co-energy principle requires no information of the load connected to the secondary winding. This indicates that the AC current distribution among parallel-connected windings is not affected by the load. Therefore, the optimum winding turn allocation can be utilized for high frequency power transformer design regardless to the load impedance.

This simple example is based on a planar transformer of an ungapped core with flat and uniformly wound winding layers, which is well-known to be effective for modeling according to the Dowell model [18]–[25]. This model is proven to be practically effective for analytical estimation of the magnetic field distribution inside the magnetic structure. Therefore, we can straightforwardly formulate the magnetic co-energy as the function of the winding current. Certainly, this type of transformers is the most basic and simple target for application of the proposed method, which is based on the extremum co-energy principle. However, as discussed in the previous section, the extremum co-energy principle is a universal insight, which is applicable to many other types of magnetic structures. As discussed in [13], the extremum co-energy was formulated using the magnetic circuit model, which can be more universally applied to many magnetic structures than the Dowell model. Therefore, the proposed method can probably be utilized universally as far as the magnetic co-energy is formulated analytically.

## Experiment

Experiments were carried out to verify homogenization of the AC current distribution using the extremum co-energy principle. In these experiments, we measured the current distribution among parallel-connected windings and the parasitic AC resistance caused by the copper loss in the transformers with the structure of Fig.2. The photograph of this transformer is presented in Fig.3. The photographs of the primary and secondary windings in this experimental transformer are presented in Fig. 4. In order to meet the assumption employed in the theory of the previous section, the primary windings were made of twisted wire strands, whereas the secondary winding was made of the copper plate. The primary windings have 14 turns, whereas the secondary winding has only one turn. The primary windings 1 and 2 were designed to have the same number of turns to be connected in parallel. Hence, the sum of the number of turns of winding layers A and D is always kept the same as that of winding layers B and C, i.e.  $N_A+N_D=N_B+N_C=14$ . The specifications of this transformer are shown in Table 1.

#### AC current distribution among parallel connected wires

First, we evaluated the current in primary windings 1 and 2 at various turns ratio  $\alpha$  (= $N_A/N_B$ ), and investigated the dependence of the current ratio on  $\alpha$ . In this experiment, we connected a load resistor to the secondary winding as shown in Fig. 4 (This load resistance was set at 0.0 $\Omega$ , 250m $\Omega$ , 500m $\Omega$  and 1.0 $\Omega$ .) and applied the AC voltage to the primary windings. Then, we observed the AC current in windings 1 and 2. The frequency of the AC voltage is set at 100kHz. The effective value of total primary current *i*<sub>total</sub> was set at 0.5Arms when the load resistance was 0.0 $\Omega$ , at 0.2Arms when the load resistance was 250m $\Omega$  and 500m $\Omega$ , and at 0.1Arms when the load resistance was 1.0 $\Omega$ .

Figures 5 and 6 show the measurement results of the effective value and the phase angle of the AC

current in primary windings 1 and 2. Figure 5(a) shows the effective value of  $i_1$  normalized with respect to the effective value of  $i_{total}$ , whereas Fig. 5(b) shows the effective value of  $i_2$  normalized with respect to the effective value of  $i_{total}$ . In addition, Fig. 6 shows the phase angle difference between  $i_1$  and  $i_{total}$  and between  $i_2$  and  $i_{total}$ . As can be seen in these figures, the experimental results well agreed with the theoretical prediction made by (10), although we adopted rough approximations in formulating the magnetic co-energy of the transformer in the theoretical analysis. Furthermore, the experimental primary current ratios of  $i_1/i_{total}$  and  $i_2/i_{total}$  were found to be scarcely affected by the load resistance, which is also consistent with the theory.

According to the theory, all the AC current will flow in primary winding 1, if  $N_A$  and  $N_D$  are both set at 7 ( $\alpha$ =1.0). Contrarily, the current will be distributed almost homogeneously, if  $N_A$  and  $N_D$  are set at 9 and 5, respectively ( $\alpha$ =1.29). As can be seen in Fig. 5, the experiment revealed that almost all the primary current flowed in primary winding 1, when  $\alpha$ =1.0. Furthermore, approximately 60% of the primary current flowed in primary winding 1, when  $\alpha$ =1.29. Therefore, this experiment supported the



Fig. 5: Experimental result of current distribution in primary windings







Fig. 7: Measurement system for the parasitic AC resistance

effectiveness of the extremum co-energy principle for designing the winding turn allocation to homogenize the AC current distribution among the parallel-connected windings.

#### Parasitic AC resistance caused by copper loss

Next, we carried out an experiment to verify minimization of the copper loss based on the optimum winding turn allocation using the extremum co-energy principle. For this purpose, we compared the measured parasitic AC resistance of the experimental transformer with the estimated parasitic AC resistance based on the theoretical current distribution.

The measured parasitic AC resistance was evaluated by measuring the AC resistance of the primary winding, while keeping the secondary winding short-circuited. Figure 7 illustrates the measurement method. An AC voltage source was connected to the primary windings to apply the AC current of 1 Arms, 100kHz. Then, the voltage and the current of the primary windings were measured using a voltage probe, a current probe, and an oscilloscope. If we denote the effective value of the voltage and the current of the primary windings as  $v_p$  and  $i_p$ , respectively, the measured parasitic AC resistance  $R_{mes}$  can be obtained as

$$R_{mes} = \frac{v_p}{i_p} \cos\theta, \tag{12}$$

where  $\theta$  is the phase angle difference between the measured voltage and current. Note that the resistance  $R_{mes}$  is measured under no flux induction because of the shorted secondary winding. Therefore, no iron loss but only the copper loss contributes to  $R_{mes}$ .

On the other hand, the estimated parasitic AC resistance was obtained by estimating the total copper loss of the primary and secondary windings. First, we measured the parasitic AC resistance  $R_p$  of the winding wire used for the primary winding in advance. The AC resistance of the winding wire was measured at 100kHz and at the same length as one of the primary windings of the experimental transformer. Second, we estimated the parasitic AC resistance of the secondary winding. As discussed in the previous work [12], the AC current flows only on the bottom surface of the copper plate. In this case, the parasitic AC resistance of the secondary winding is identical to the resistance of the copper plate with the thickness equal to the skin depth. Hence, the parasitic AC resistance  $R_s$  of the secondary winding was estimated as

$$R_s = \rho \frac{\pi (r_b + r_a)}{\delta (r_b - r_a)},\tag{13}$$

where  $\rho$  is the resistivity of the copper;  $r_a$  and  $r_b$  are the inner and outer diameter of the copper plate for the secondary winding, respectively. The symbol  $\delta$  is the skin depth calculated as

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}},\tag{14}$$



Fig. 8: Comparison of parasitic AC resistance between of measurement and theoretical estimation

where  $\omega$  is the angular frequency and  $\mu$  is the permeability of the copper. Third, the total copper loss  $P_{copper}$  was estimated as follows based on  $R_p$ , i.e. the parasitic AC resistance of one primary winding wire measured in the first step, and  $R_s$ , i.e. the parasitic AC resistance of the secondary winding estimated in the second step:

$$P_{copper} = R_p \left\{ \kappa^2 + (1 - \kappa)^2 \right\}_p^2 + R_s i_s^2 = \left\{ \left( 2\kappa^2 - 2\kappa + 1 \right) R_p + n^2 R_s \right\}_p^2,$$
(15)

where  $i_p$  and  $i_s$  are the effective values of the primary and secondary current, respectively; *n* is the turns ratio between the primary and secondary windings. Finally, the estimated AC resistance  $R_{est}$  was obtained using (11) and (15) as

$$R_{est} = (2\kappa^2 - 2\kappa + 1)R_p + n^2R_s = \left\{ 2\left(\frac{5 - 3\alpha}{3\alpha^2 - 6\alpha + 5}\right)^2 - 2\left(\frac{5 - 3\alpha}{3\alpha^2 - 6\alpha + 5}\right) + 1 \right\}R_p + n^2R_s.$$
(16)

Figure 8 shows the comparison result of the measured AC resistance  $R_{mes}$  and the estimated AC resistance  $R_{est}$ . The result revealed that the measured AC resistance was almost consistent with the theoretical estimation. According to the theory, the primary current is split evenly, when  $\alpha$ =1.29. Therefore, the parasitic AC resistance of the experimental transformer is predicted to take the minimum at  $\alpha$ =1.29. This prediction was verified by the experimental result, because the measured AC resistance was found to take the minimum at  $\alpha$ =1.29. Consequently, the experimental results successfully supported that the extremum co-energy principle can be effectively applied to the optimum design of the winding turn allocation for minimizing the copper loss of high frequency power transformers.

#### Conclusions

Parallel-connected windings are widely utilized for the primary and secondary windings of high frequency power transformers to reduce the copper loss. However, the AC current may be concentrated in one winding of the parallel-connected windings, hindering the reduction effect of the copper loss. In order to address this issue, this paper proposed application of the extremum co-energy principle to transformer design. This paper presented a design optimization method of winding turn allocation among the winding layers. The extremum co-energy principle was found to offer comparatively straightforward calculation for predicting the winding turn allocation that homogenizes the AC current distribution among parallel-connected windings. Furthermore, the experiment supported appropriateness of this optimization method. Consequently, the extremum co-energy principle is concluded to be a promising tool for design optimization of parallel-connected windings of high frequency power transformers.

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