Feasibility Study of Fractal-Fin Heat Sink for Improving Cooling Performance of Switching Power Converters

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***Abstract*—Recent progress of the semiconductor switching devices enabled the high-frequency design of the switching power converters, which successfully led to the miniaturization of the circuit elements. However, the heat sink has scarcely been miniaturized during past decades. Now, the heat sinks are one of the major obstacles that hinder further size reduction of the switching power converters. Conventionally, the heat sinks are designed to have a number of flat aluminum fins extending from the aluminum base. However, these flat fins tend to make the laminar airflow passing through the fins. This laminar airflow keeps the air contacting the fin surface to remain on the surface. As a result, the fin surface is covered with the air already heated in the upstream and the fresh open air is prevented from contacting with the fin surface, thus deteriorating the cooling performance of the heat sinks. To overcome this issue, this paper proposes a heat sink structure with a novel fin geometry. The proposed fin geometry incorporates two sizes of the louvers, which are disposed to form a fractal-like pattern. These louvers are intended to break the laminar flow on the fin surface, thus promoting the mixing of the air to improve the cooling performance. FEM simulation of the heat transfer was carried out to verify this concept. The result showed improvement in the heat transfer coefficient, implying the effectiveness of the proposed structure for miniaturization of the heat sink.**

Keywords— cooling, heat sink, fractal, fin geometry, heat transfer

I. Introduction

Recent progress of the semiconductor switching devices has given rises to the high-frequency switching power converters owing to their fast switching capability. High-frequency operation of the switching power converters can miniaturize the passive components such as the inductors and the capacitors. Therefore, the recent power converters have been greatly miniaturized, which expanded the application of the switching power converters in the field of telecommunication equipment.

Compared with the successful miniaturization of the circuit elements by the high-frequency operation, the heat sinks have been scarcely miniaturized during past decades. As a result, the heat sinks are now recognized as one of the major volume contributors in many cutting-edge high-frequency power converters [1]–[7]. Furthermore, the recent power converters may need an even more efficient cooling method because the circuit elements tend to have a small surface area for the heat dissipation as a result of miniaturization by the high-frequency operation. Therefore, improving the cooling performance within the limited volume is an essential topic to be addressed for further miniaturization of the switching power converters.

Conventionally, the heat sinks are comprised of a number of flat aluminum fins extending in perpendicular to the base plate [5][8][9], as exemplified in Fig. 1(a). This structure is effective for increasing the surface area contacting with the air. However, according to the fluid dynamics, the air contacting the fin surface continues to remain near the surface in this conventional structure, forming a thick boundary layer [10] on the fin surface. As a result, the fin surface is covered with the air already warmed in the upstream, thus preventing the fresh open air from contacting the fin surface. Therefore, the conventional heat sink tends to show inefficient heat dissipation particularly at the fin surface positioned in the downstream of the airflow, thus resulting in inefficient cooling in spite of its large surface area.

The purpose of this paper is to propose a heat sink structure with a better cooling performance by solving the aforementioned problem of the conventional heat sink structure. With regard to the solution of this problem, a fractal geometry has been recently pointed out to be effective for improving the heat dissipation [11]–[15]. Some of the preceding studies [12], [14], [15] have investigated the effectiveness of the fractal geometry for application to the heat sinks and suggested improvement in the cooling performance.

Particularly, a preceding study [15] has even experimentally proven the improvement in the heat dissipation by introducing the fractal geometry to the heat sink structure. However, this experimental heat sink prototype

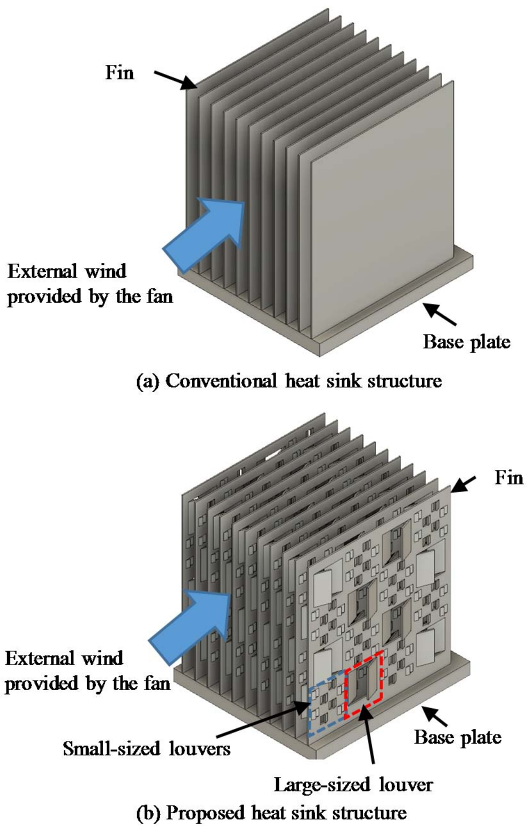


Fig. 1. Conventional and proposed heat sink structures.

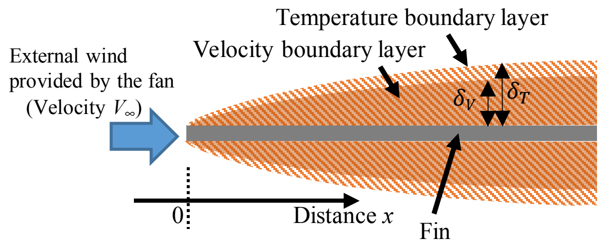


Fig. 2. Velocity and temperature boundary layer formed on a fin surface of the conventional heat sink structure by the external wind provided by the fan. (Horizontal sectional view)

presented in [15] was designed only for evaluation of the principle and is not suitable for practical application to power converters. In fact, this prototype of the preceding study is made of a number of small fins dispersed in the space to form the Sierpinski gasket, which is hardly convenient for industrial manufacturing. Therefore, this paper rather bases our investigation on a heat sink structure comprised of a base plate and a number of fins, similarly as in the conventional heat sink structure, and investigates the improvement in the cooling performance by introducing the fractal geometry to the fin surface.

This paper elucidates the probability of the cooling performance improvement by the proposed heat sink structure using the FEM-based heat transfer simulation. Section II presents the proposed heat sink structure and theoretically analyzes the mechanism of the possible improvement in the cooling performance. Then, section III presents the simulation results to evaluate the theoretical analysis. Finally, section IV gives the conclusions.

II. PROPOSED HEAT SINK STRUCTURE

A. Structure Overview

Figure 1 illustrates the proposed and conventional heat sink structures. Both of the structures have the aluminum base plate and the fins vertically extending from the plate. The fins are plain flat aluminum plates in the conventional structure, whereas the fins have the small-sized and large-sized louvers in the proposed structure. These louvers are made by making the H-shape cut on the fin and then lifting up one flap while pushing down the other flap.

The large-sized louvers are placed to form the checkered pattern on the fin surface. The small-sized louvers are also placed to form the checkered pattern in the square area surrounded by the large-sized louvers. Hence, the small-sized louvers have the quarter size of the large-sized louvers.

The fins of the proposed heat sink structure are alternately turned over so that each large-sized louver on a fin is faced with a checkered pattern of the small-sized louvers on the neighboring fins. This feature is similar to the previously reported heat sink structure with fractal geometry [15]. The large-sized and small-sized louvers of the proposed structure correspond to the large-sized and small-sized pits, respectively, of the previously reported structure. In this sense, the proposed heat sink structure also has the fractal geometry as in the previously reported structure.

The louvers of the proposed heat sinks are made without cutting off any part of the fins. Therefore, the proposed heat sink structure has the same surface area as the conventional structure except for the cut section added by the H-shape cut. This indicates that the difference in the cooling performance between the proposed and conventional structures can be mainly attributed to the fin geometry.

The proposed heat sink structure can improve the cooling performance due to its fractal geometry of the fin. The large-sized louvers are implemented to make large-sized vacant space on the fin surface, whereas the small-sized louvers are implemented to make small-sized vacant space. These two sizes of the vacant space contribute to improving the cooling performance. The following subsections discuss the reason based on the fluid dynamics.

B. Cooling Mechanism in Conventional Heat Sink Structure

We start the discussion from the cooling mechanism of the conventional heat sink structure. We assume that the fan is attached to the heat sink and that this fan provides the uniform external wind along the fin surface as shown in Fig. 2. In the case of no fans to provide the wind, the similar discussion can be also applied by assuming the vertical updraft along the fin surface generated by the buoyancy in the natural convection.

According to the fluid dynamics, the wind velocity at the fin surface must be zero. Therefore, the wind velocity must greatly change from zero to the velocity of the external wind, as the distance from the fin surface increases. This change occurs in a thin layer contacting the fin surface, which is commonly called the velocity boundary layer.

The thickness of the velocity boundary layer is dependent on the distance x from the front edge of the fin, i.e. the edge that facing the external wind. The thickness increases as the distance x become larger. Theoretically, the order of the thickness δ_V is given as

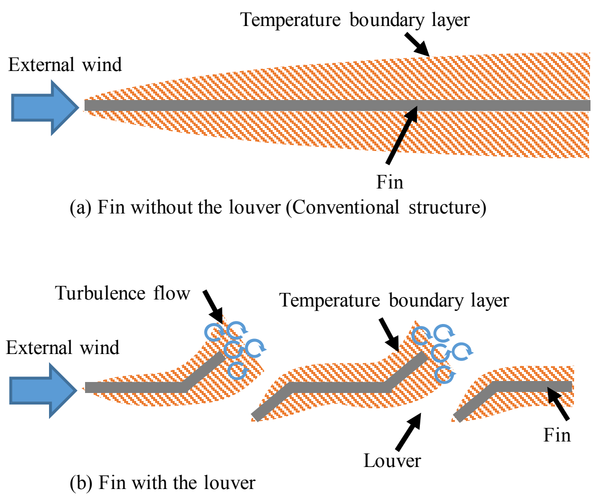


Fig. 3. Temperature boundary layer formed on a fin with and without the louvers. (Horizontal sectional view)

$$\delta_V \approx 5.0 \Big/ \sqrt{\frac{V_\infty}{\upsilon x}}, \tag{1}$$

where V_∞ is the velocity of the external uniform wind provided by the fan and υ is the kinetic viscosity. Hence, the thickness δ_V is proportional to the square of x.

Similarly, as the wind velocity, the temperature also changes greatly from the fin surface to the ambient temperature within a thin layer contacting the fin surface. This layer is called the temperature boundary layer. The thickness of this layer (δ_T) is known to be proportional to that of the velocity boundary layer (δ_V). According to the fluid dynamics [10], the ratio between δ_T and δ_V is approximately given as

$$\frac{\delta_T}{\delta_V} \approx \frac{1}{1.026\sqrt[3]{\mathrm{Pr}}}. \tag{2}$$

The symbol Pr is the Prandtl number defined as the ratio of the kinetic viscosity υ to the thermal diffusivity α. Therefore, Pr is given as

$$\mathrm{Pr} = \frac{\upsilon}{\alpha} = \frac{\upsilon \rho c_p}{k}, \tag{3}$$

where ρ is the density of the fluid, c_p is the specific heat at constant pressure (per unit mass), and k is the thermal conductivity. Hence, the Prandtl number of the air is calculated to be approximately 0.7; and therefore, the ratio δ_T/δ_V is approximately 1.1.

The heat transferred from the unit surface area of the fin is proportional to the temperature difference between the fin and the ambient. The proportionality coefficient of this relation, called the local heat transfer coefficient h_l, can be obtained using δ_T. According to the fluid dynamics, the local heat transfer coefficient h_l is given as

$$h_l = \frac{3k}{2\delta_T} \approx \frac{1.026\sqrt[3]{\mathrm{Pr}}}{5.0} \frac{3k}{2} \sqrt{\frac{V_\infty}{\upsilon x}}, \tag{4}$$

As can be seen in (4), the increasing thickness of the thermal boundary layer hinders the heat transfer from the fin; and therefore, the local heat transfer coefficient decreases as x increases. This result indicates that the heat sink with large fin width tends to have low local heat transfer coefficient. The low heat transfer coefficient indicates that the fins must have large temperature difference from the ambient to dissipate the same amount of heat under the same fin surface area. Therefore, a large-sized fin is less effective in the cooling than a number of small-sized fins, if compared under the same total fin surface area.

C. Cooling Mechanism in Proposed Heat Sink Structure

As we have seen, the thickness of the temperature boundary layer is one of the important factors that determine the cooling performance. This layer separates the fresh open air from the fin surface, and the heat is dissipated from the fin surface through this layer. Therefore, the thick temperature boundary layer needs a large temperature difference from the ambient to dissipate the heat, which indicates bad cooling performance.

However, the temperature boundary layer grows thick, as the air flows downwards. Therefore, frequently extinguishing this layer may be an effective method to keep this layer thin, leading to better cooling performance. For this purpose, the proposed structure incorporates various louvers to make the discontinuity on the fin surface without decreasing the fin surface area. This discontinuity can break and rip off the thermal boundary layer from the fin surface as depicted in Fig. 3. Furthermore, breaking the thermal boundary layer causes turbulence flow at the tail of the layer. This turbulence flow mixes the heated air in the thermal boundary layer with the fresh air provided by the external wind, thus finally extinguishing the thermal boundary layer.

The small-sized louvers may seem to be more effective than the large-sized louvers to frequently break the thermal boundary layer because the small-sized louvers can be implemented in a larger number on the fin surface. However, the discontinuity made by the small-sized louvers has the smaller scale than the large-sized louvers. Therefore, small-sized louvers may be insufficient to break the thermal boundary layer, particularly if the scale of the discontinuity is smaller than the thermal boundary layer. Furthermore, small-sized louvers tend to have less mixing capability of the air than the large-sized louvers because the discontinuity added by the louvers can mix the air within the scale of the discontinuity.

Consequently, both of the large-sized and small-sized louvers have their own merits and drawbacks. The large-sized louvers can effectively break the boundary layer and mix the air in the large scale, although the number of the louvers is limited due to its size. On the other hand, the small-sized louvers can frequently break the boundary layer, although the ability to break the boundary layer is less and the mixing scale is smaller than those of the large-sized louvers.

The proposed structure has both of the small-sized and large-sized louvers. In the proposed structure, both of the small-sized and large-sized louvers collaborate to improve the cooling performance. The small-sized louvers frequently break the boundary layer and mix the air of the boundary layer with the neighboring air frequently but in a smaller scale. Then, the large-sized louvers can further mix this partially mixed air with the fresh air on a larger scale, thus effectively promoting the mixing process to extinguish the thermal boundary layer.

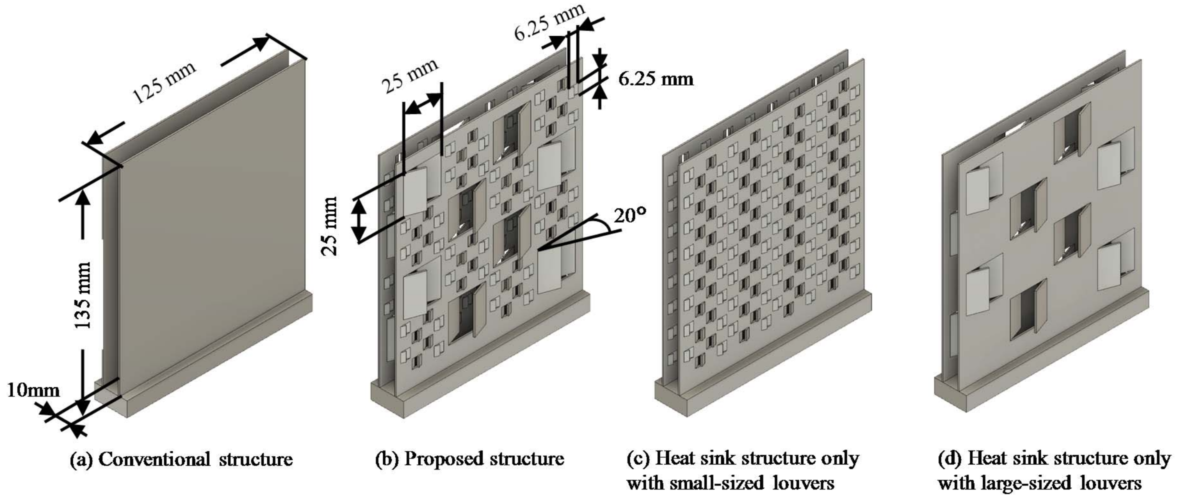


Fig. 4. Simulation models of the 4 types of the heat sinks.

Consequently, the proposed heat sink structure can be expected to improve the cooling performance.

III. SIMULATION

A. Simulation Models

The effectiveness of the proposed heat sink structure was evaluated by simulation. We utilized COMSOL Multiphysics 5.3 as the simulator. In this simulation, we compared the heat dissipation among 4 types of the heat sinks, as illustrated in Fig. 4. Fig. 4(a) is the conventional heat sink structure. Figure 4(b) is the proposed heat sink structure. Figure 4(c) is the heat sink structure with only small-sized louvers on the fin surface. Figure 4(d) is the heat sink structure with only large-sized louvers. The fins of these structures can be made from the flat square aluminum plate. Therefore, the surface area of these fins is almost the same. Actually, the fin surface area of Fig. 4(b), Fin 4(c), and Fig. 4(d) are 107%, 109%, and 102% of that of Fig. 4(a), respectively.

Models that incorporate many fins requires enormous calculation time. Therefore, in this simulation, only two fins standing on the base plate are constructed in the model space and two sides of the model space were set at the periodic boundary condition, as illustrated in Fig. 5. This setting is equivalent to having an infinitely large number of fins. The upper and bottom boundaries were set at the rigid non-slip and thermally insulated boundary condition. We set the bottom boundary at the top of the base plate so that the external wind is not affected by the base plate. The front and bottom boundary is the inlet and outlet of the wind. The front boundary was set at a uniform and constant wind velocity at the temperature 293K. In this simulation, the inlet wind velocity was varied from 0.2m/s to 4.0m/s. On the other hand, the outlet boundary condition was set at the constant pressure 1013hPa.

The heat sinks were all made of aluminum with the thermal conductivity of 225W/m·K. The thickness of the fins was set at 1mm; and the width and height of the fins were set at 125mm and 135mm, respectively. The base plate has the height of 10mm. Because this base plate is assumed to be attached to the electric components in practical applications, we gave the uniform and constant heat to the bottom surface of the base plate as a heat source. We set the heat given to the bottom surface of the base plate at 3000W/m^2. The specifications of the simulation are summarized in Table I.

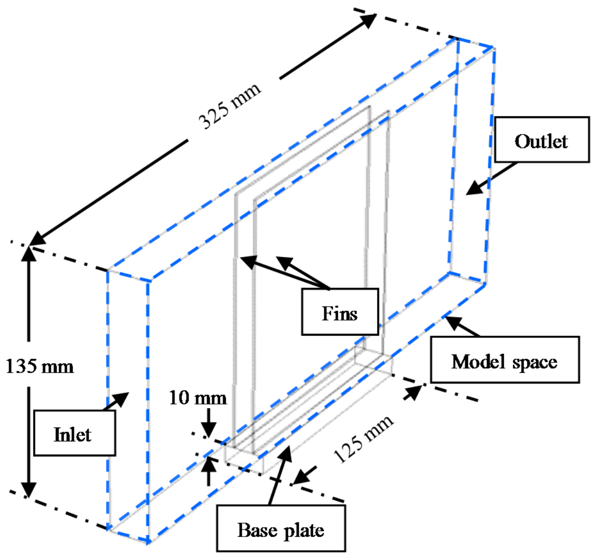


Fig. 5. Model space of the simulation.

B. Thermal Resistance

Firstly, we compared the thermal resistance of the heat sink among the models. The thermal resistance is a common indicator of the cooling performance of a heat sink among many electronics researchers. The thermal resistance of a heat sink R_{sa} is the necessary temperature difference from the

TABLE I. SPECIFICATIONS OF SIMULATION,

Inlet	External wind velocity	0.2–4m/s
	External air temperature	293K
Outlet	Pressure	1013hPa
Side boundary	Periodic boundary condition	
Upper & bottom boundary	Non-slip & insulation condition	
Base plate	Heat source	3000 W/m^2
Turbulence model	L-VEL	

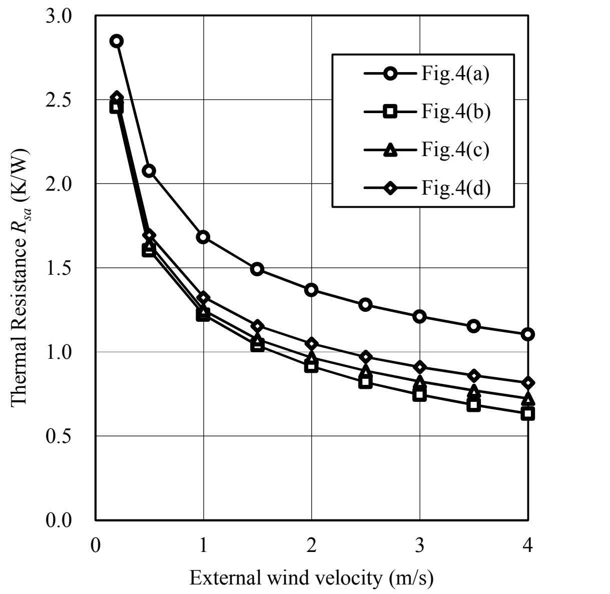


Fig. 6. Simulation results of the thermal resistance of the heat sink R_{sa}.

ambient, i.e. the external wind, to dissipate the unit amount of the heat. Hence, the thermal resistance R_{sa} is defined as

$$R_{sa} = \frac{T_b - T_0}{Q}, \tag{5}$$

where T_b is the temperature of the base plate, T_0 is the temperature of the external wind, and Q is the total heat given to the base plate. The thermal resistance is the inverse value of the heat transfer coefficient. Therefore, the thermal resistance takes a small value, if the heat sink shows excellent cooling performance.

Figure 6 presents the simulated result of the thermal resistance of the 4 models. The proposed structure showed the smallest thermal resistance regardless of the external wind velocity. The proposed heat sink structure even showed a smaller thermal resistance than the structures with either the large-sized or small-sized louvers. This is consistent with the theoretical discussion because the result indicates that both of the large-sized and small-sized louvers collaboratively contributed to the improvement in the cooling performance.

The similar features were also found in the previously-reported heat sink structure. Therefore, this result successfully supports the feasibility of the fractal geometry of the fin, although this proposed structure has a more practical structure than the previously-reported structure.

In general, the heat dissipation from the heat sinks contains two processes: 1. The thermal conduction from the bottom surface of the base plate to the fin, and 2 the heat transfer from the fin surface to the external wind. As a result, the thermal resistance of a heat sink can be expressed as a sum of the thermal resistance each of which represents one of these two processes:

$$R_{sa} = \frac{T_b - T_f}{Q} + \frac{T_f - T_0}{Q} = R_{sf} + R_{fa}, \tag{6}$$

where T_f is the average temperature of the fin; R_{sf} is the representative thermal resistance from the bottom surface of

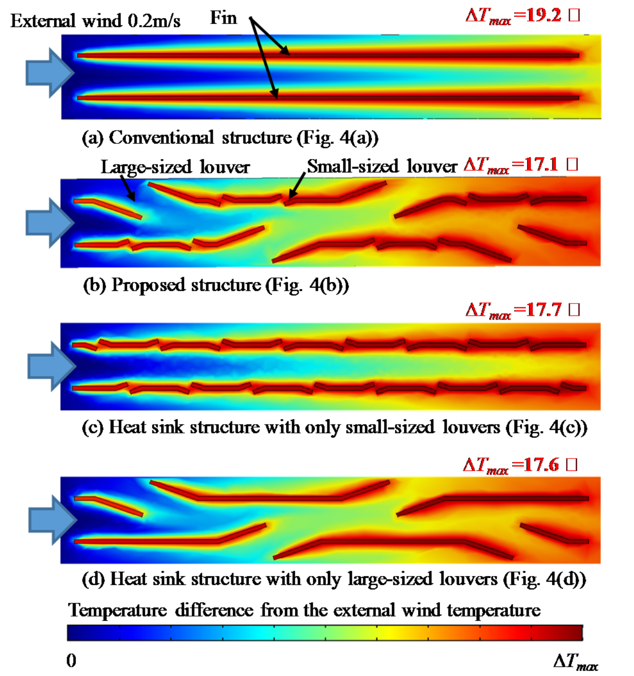


Fig. 7. Temperature difference of the air and fins from the external wind in the horizontal cross-section of the model space when the external wind velocity was set at 0.2m/s. For each model, the temperature difference is normalized by the maximum temperature difference in the cross-section (ΔT_{max}).

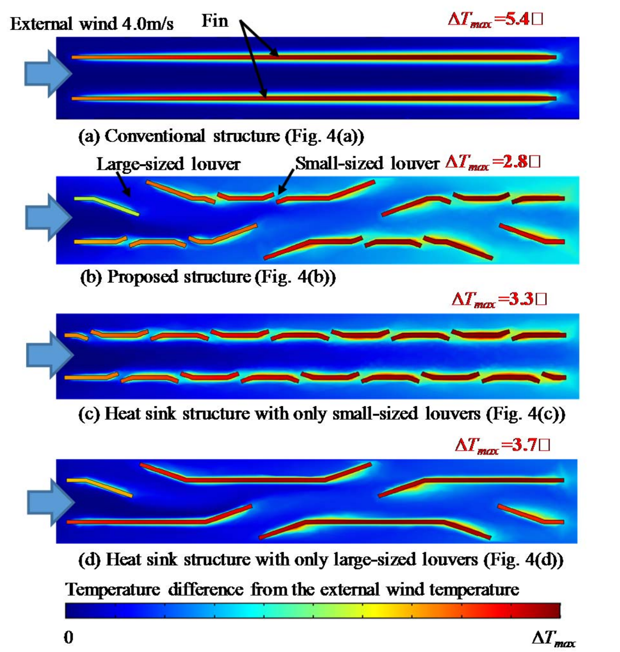


Fig. 8. Temperature difference of the air and fins from the external wind in the horizontal cross-section of the model space when the external wind velocity was set at 4.0m/s. For each model, the temperature difference is normalized by the maximum temperature difference in the cross-section (ΔT_{max}).

the base plate to the fin surface, defined as R_{sf}=(T_b–T_f)/Q; and R_{fa} is the representative thermal resistance from the fin surface to the external wind, defined as R_{fa}=(T_f–T_0)/Q.

According to the theoretical discussion in the previous section, the proposed structure is expected to be effective for reducing R_{fa}, because the thermal boundary layer affects only R_{fa}. Therefore, the subsequent subsection investigates R_{sf} and

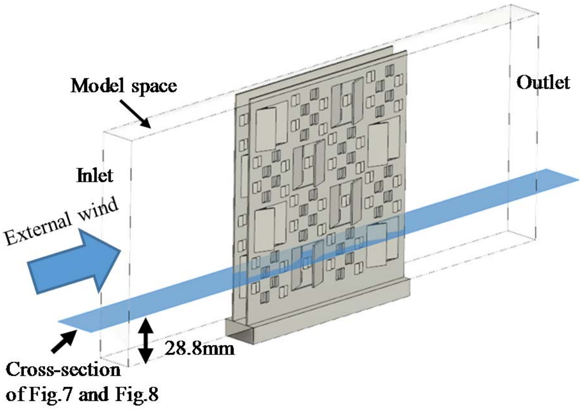


Fig. 9. Horizontal cross-section taken for Fig. 7 and Fig. 8.

R_{fa}, as well as the thermal boundary layer, to validate that the difference in the thermal boundary layer actually resulted in the reduction of R_{sa}.

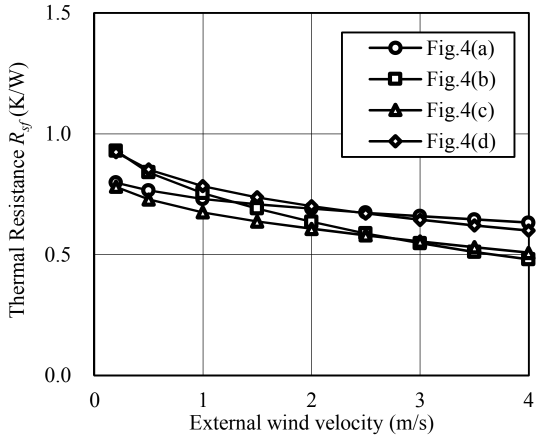
C. Temperature Boundary Layer

Figures 7 and 8 compare the horizontal cross-sectional view of the temperature difference from the external wind provided through the inlet. (The temperature difference is normalized by the maximum temperature difference in the cross-section.) The cross-section was taken at the height of 28.8mm from the top of the base plate, as shown in Fig. 9. Figure 7 shows the temperature difference distribution when the external wind velocity was set at 0.2m/s. On the other hand, Fig. 8 shows the temperature difference distribution when the external wind velocity was set at 4.0m/s.

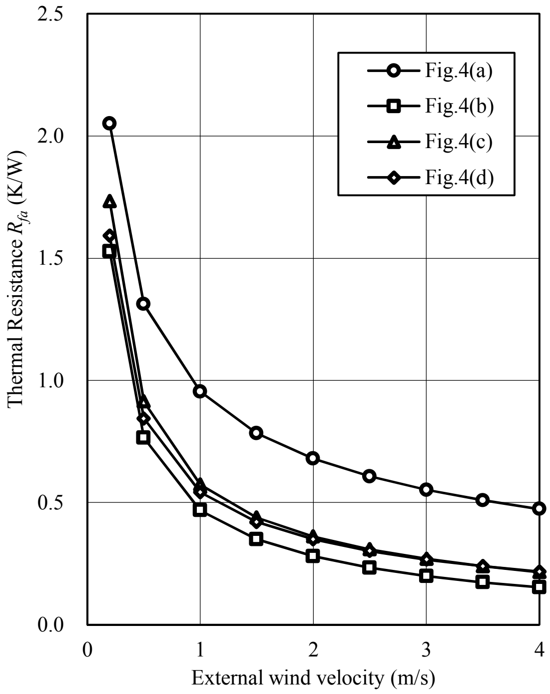
As can be seen in Fig. 7(a) and Fig. 8(a), the conventional structure clearly showed the temperature boundary layer. The temperature boundary layer is the thin layer surrounding the fin surface, in which the temperature increases greatly. The thickness of the temperature boundary layer grows as the wind goes downwards, as is expected from the theory. Separated by the thick boundary layer, the air outside the thermal boundary layer remains almost constant temperature, implying the inefficient heat dissipation from the fin surface.

The heat sink structure only with the large-sized louvers, i.e. Fig. 4(d), showed the successful break and extinction of the temperature boundary layer at the louvers, although the breakpoints of the boundary layer is a few. On the other hand, the heat sink structure only with small-sized louvers, i.e. Fig. 4(c), broke the temperature boundary layer more frequently. Nonetheless, the temperature boundary layer is extinguished insufficiently at the louvers because the flaps of the louvers were sunk in the layer, particularly when the wind velocity is small.

The proposed structure, having both of the large-sized and small-sized louvers, showed both the features of Fig. 4(c) and Fig. 4(d). In the simulation results of the proposed structure, the small-sized louvers mixed the air in a small scale; and then, the large-sized louvers mixed the partially mixed air in a larger scale. Therefore, the combination of these two levels of mixing successfully occurred, as is expected from the theory. Certainly, Fig. 7(b) did not show significant improvement in the mixing of the warmed air with the ambient air compared with Fig. 7(c) and Fig. 7(d). However, Fig. 8(b) showed the spread of the warmed air near the fins into the wider region than Fig. 8(c) and Fig. 8(d), in the downstream. This feature implies that the two levels of the mixing can improve the heat dissipation particularly when the external wind velocity is large.



(a) Thermal Resistance R_{sf}



(b) Thermal Resistance R_{fa}

Fig. 10. Simulation results of the thermal resistance from the botton plate to the fin surface R_{sf} and the thermal resistrance from the fin surface to the external wind R_{fa}.

D. Thermal Resistance R_{sf} and R_{fa}

In order to evaluate the effect of the aforementioned difference in the thermal boundary layer on the heat transfer, we evaluated R_{sf}, i.e. the thermal resistance from the base plate to the fin surface, and R_{fa}, the thermal resistance from the fin surface to the external wind.

Figure 10 shows the simulation result of R_{sf} and R_{fa}. The proposed structure, i.e. Fig. 4(b), showed the smallest R_{fa} among the models, whereas the conventional structure, i.e. Fig. 4(a), showed the largest R_{fa}. This feature was similar to the result of R_{sa}, shown in Fig. 6. Furthermore, the proposed heat sink structure showed far smaller R_{fa} than Fig. 4(c) and Fig. 4(d), particularly when the external wind velocity is large. These features support that the difference in the total thermal resistance of the heat sink, i.e. Fig. 6, is contributed by the difference in the thermal boundary layer.

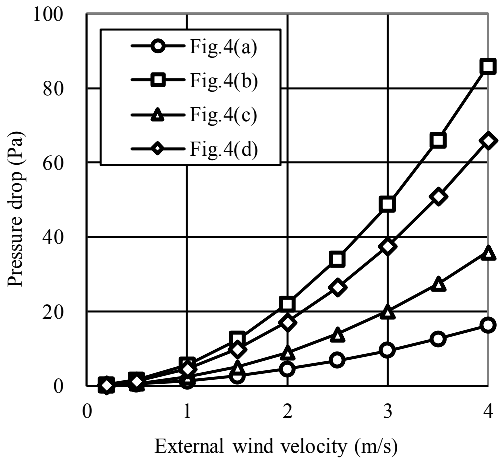


Fig. 11. Simulation results of the pressure drop by the heat sink.

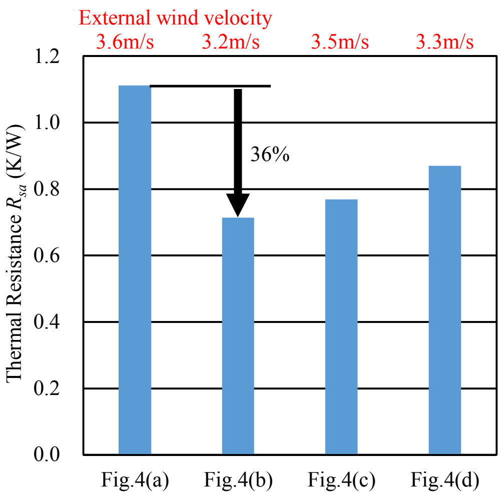


Fig. 12. Estimation result of the thermal resistance of the heat sinks provided with the external wind using the commercial DC fan.

On the other hand, that the 4 heat sink structures showed similar R_{sf}. In fact, R_{sf} of the 4 heat sink structures ranged from 87% to 114% of the average value of the 4 structures when the external wind velocity was 4.0m/s. As for the other external wind velocity, the variation in R_f was even smaller. Therefore, R_{sf} scarcely explains the difference in the total thermal resistance, i.e. R_{sa}, between the conventional and proposed structures.

E. Pressure Drop

Finally, we evaluated the pressure drop. The louvers of the proposed structure tend to slightly hinder the air flow along the fins. As a result, the proposed structure needs to add more pressure than the conventional structure for applying the same external wind velocity. In other words, the proposed structure has smaller external wind velocity than the conventional structure, if compared under the same fan. Therefore, evaluating the pressure drop is important because large pressure drop may results in the significant reduction in the external wind velocity, thus deteriorating the cooling performance.

Figure 11 presents the simulation result of the pressure drop of the 4 structures. The heat sinks with the louvers showed far greater pressure drop than the conventional structure. Particularly, the proposed heat sink structure showed more than 4 times as large pressure drop as the conventional heat sink structure, when the external wind velocity is greater than 1.0m/s.

In order to investigate the possible deterioration of the cooling performance, we estimated the effect of this pressure drop on the thermal resistance of the heat sink R_{sa}. We calculated R_{sa} under the assumption that the inlet of the heat sinks is covered with an array of commercially available DC fans. We adopted 9GV1212P1G01 (Sanyo Denki) supplied with 12Vdc as the fans for the estimation.

Figure 12 shows the estimation result of the thermal resistance as well as the external wind velocity. In spite of the difference in the pressure drop among the 4 heat sink structures, the external wind velocity was almost the same. Actually, the external wind velocity ranged from 94% to 106% of the average value of the 4 heat sink structures. As a result, the proposed structure showed the smallest thermal resistance among the 4 structures. The proposed structure reduced the thermal resistance by 36% compared with the conventional structure. Consequently, the pressure drop of the proposed structure did not lead to the significant deterioration of the improvement in the cooling performance.

IV. Conclusions

Recently, the heat sinks are recognized as one of the major contributors to the volume of the power converters. However, the conventional heat sink structure suffers from inefficient cooling because the fins tend to have the thick thermal boundary layer, which hinders the heat dissipation from the fin to the air. In order to mitigate this issue, this paper proposed a novel heat sink structure. This structure has fins extending from the base plate, similarly to the conventional structure. Nonetheless, the fins of the proposed structure have a fractal geometry to break the thermal boundary layer. Basic principles of this proposed structure were examined by FEM simulation. The result showed remarkable improvement of the cooling performance in the proposed structure by breaking the thermal boundary layer, suggesting the feasibility of the proposed structure for possible miniaturization of the cooling system of power converters.

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