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A Simple Behavior Model for Switched Reluctance Motors Based on Magnetic Energy

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Abstract—A number of analytical models for switched reluctance motors have been proposed to promote development of control techniques that can alleviate the torque and source current ripples. However, these models can suffer from a large database of the nonlinearity as well as complicated derivation process of the torque and the voltage-current relation. The purpose of this paper is to propose a simple practical behavior model with small database and straightforward derivation of the torque and the voltage-current relation. The proposed model has a simple database of the magnetic energy as a matrix. The flux linkage and the electrical angle are chosen as the state variables. Along with theoretical formulation of the model, this paper presents a practical method of the model construction. In addition, experiments successfully predicted both torque and current waveforms, supporting appropriateness of the proposed model.

Keywords—switched reluctance motor; analytical model; magnetic energy; Lagrangian

I. INTRODUCTION

Switched reluctance motors (SRMs) are attracting increasing attention for their robust mechanical construction and cost-effectiveness [1]–[3], [5], [6]. However, their industrial applications can be limited by large torque ripple and source current ripple caused by severe mechanical and electrical nonlinearity [4]–[6]. Therefore, motor control techniques are energetically studied to alleviate these ripples.

Development of control techniques generally requires analytical motor models as the theoretical basis. Particularly, simple models are preferable for discovering simple solutions. As for permanent magnet motors, simple linear motor models have been widely utilized for practical applications [7]–[10]. On the other hand, analytical motor models for SRMs tend to be difficult because they are required to model the nonlinearity.

This issue is addressed by a number of analytical models proposed for SRMs [11]–[23]. These models are proposed to minimize the database of the nonlinearity. Many of these models [11]–[21] regard the phase current as the state variable, and formulates the flux-linkage as a function of the phase current and the electrical angle. However, this type of formulation may cause difficulties in analyzing real time behavior of SRMs, because the voltage source inverters are

generally utilized for the motor drive, and therefore the flux linkage is directly determined according to the inverter output rather than the phase current.

Another approach is to choose the flux linkage as the state variable. For example, [22][23] formulate the phase current as the function of the flux linkage and the electrical angle. Because the flux linkage can be more convenient as the state variable, this approach may be promising for practical uses. In addition, as shown in this paper, mechanical and electrical behavior of SRMs can be conveniently derived by the magnetic energy formulated as a function of the flux linkage and the electrical angle. Hence, SRMs can be effectively modeled based on the magnetic energy.

The purpose of this paper is to show how the magnetic energy can construct a simple analytical model for SRMs. Certainly, the modeling theory of the proposed model is similar to that proposed in [23]. However, calculation of the torque in [23] may be complicated because it requires the following two steps: 1. All of the phase current is integrated with respect to the flux linkage; 2. The result is differentiated with respect to the mechanical angle. As shown in subsection II.A, calculation of the torque based on the magnetic energy can be slightly convenient because it only needs differentiation with respect to the mechanical angle. In addition, this paper improved the parameter extraction method for the proposed model as discussed in Subsection II.C.

This paper derives the basic theory of the proposed model using Lagrangian dynamics. Lagrangian dynamics has been widely applied to mechanical systems. However, recent studies [24]–[34] showed that it can be also applied to power electronics. Particularly, [34] showed that Lagrangian dynamics can analyze nonlinear electrical and mechanical characteristics of SRMs. Therefore, Lagrangian dynamics can be expected to offer convenient basis to the modeling theory of SRMs.

The following discussion consists of four sections. Section II presents the modeling theory as well as the parameter extraction method for the proposed model. Section III presents the experimental verification results of the proposed model. Finally, Section IV gives conclusions.

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II. PROPOSED MODEL

A. Basic Theory of Lagrangian SRM Model

Basic theory of the proposed model can be derived according to the Lagrangian model of SRMs [34]. As shown in literature [24]–[34], the Lagrangian model is configured as a scalar function called Lagrangian, which has the same dimension as the energy.

For convenience, we assume the three-phase concentrated winding SRM as shown in Fig. 1. The motor is assumed to have multiple rotor pole pairs, and each rotor pole pair has as many stator poles as the phases. Hence, each phase consists of series-connection of as many windings as the rotor pole pairs. All windings belonging to phase U, V, and W is assumed to have the same number of turns N .

Let L_M be the Lagrangian of the motor. According to [34], L_M can be defined as

$$L_M = \frac{1}{2} I \dot{\theta}_M^2 + N \dot{q}_U \phi_U + N \dot{q}_V \phi_V + N \dot{q}_W \phi_W - E(\theta_M, \phi_U, \phi_V, \phi_W), \quad (1)$$

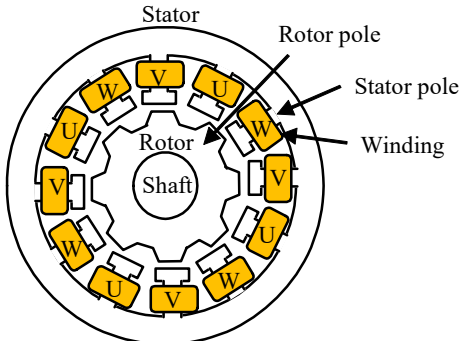
where I is the moment of inertia; θ_M is the mechanical angle; q_U , q_V , and q_W are the time integration of the current of phase U, V, and W, respectively; ϕ_U , ϕ_V , and ϕ_W are the sum of the flux interlinked with the windings in phase U, V, and W, respectively; E is the magnetic energy of the motor. The dot over a variable indicates its time derivative. Hence, \dot{q}_U , \dot{q}_V , \dot{q}_W are the phase current.

Next, we apply a simple coordinate transformation. We introduce the electrical angle θ_E and the flux linkage λ_U , λ_V , and λ_W of phase U, V, and W, defined as

$$\theta_E = P\theta_M, \quad \lambda_U = N\phi_U, \quad \lambda_V = N\phi_V, \quad \lambda_W = N\phi_W, \quad (2)$$

where P is the number of the rotor poles.

Substituting (2) into (1) yields



U, V, W: phases of the motor

Fig. 1. Example of a three-phase concentrated-winding SRM.

$$L_M = \frac{1}{2P^2} I \dot{\theta}_E^2 + \dot{q}_U \lambda_U + \dot{q}_V \lambda_V + \dot{q}_W \lambda_W - E(\theta_E, \lambda_U, \lambda_V, \lambda_W). \quad (3)$$

Now, we consider an arbitrary system including the SRM under consideration. We denote the Lagrangian and the dissipation function [29][35][36] of the system as L_A and D_A , respectively. In addition, we consider Lagrangian L'_A in which contribution of the SRM is omitted. Hence, $L_A = L'_A + L_M$. Because λ_U , λ_V , and λ_W are not contained outside the motor, λ_U , λ_V , and λ_W are not contained in L'_A and D_A . Hence, Euler-Lagrange equation [29][35][36] of L_A and D_A with respect to λ_U , λ_V , and λ_W yields $\partial L_M / \partial \lambda_U = 0$, $\partial L_M / \partial \lambda_V = 0$, and $\partial L_M / \partial \lambda_W = 0$, respectively. As a result, we obtain

$$\dot{q}_U = \frac{\partial E(\theta_E, \lambda_U, \lambda_V, \lambda_W)}{\partial \lambda_U}, \quad \dot{q}_V = \frac{\partial E(\theta_E, \lambda_U, \lambda_V, \lambda_W)}{\partial \lambda_V}, \quad (4)$$

$$\dot{q}_W = \frac{\partial E(\theta_E, \lambda_U, \lambda_V, \lambda_W)}{\partial \lambda_W}.$$

Note that (4) can be obtained for any arbitrary system. Hence, (4) gives definition of the phase current.

In addition, the torque τ is defined as the generalized force [37] with respect to θ_M . Hence, we obtain

$$\tau = \frac{\partial L_M}{\partial \theta_M} = P \frac{\partial L_M}{\partial \theta_E} = -P \frac{\partial E(\theta_E, \lambda_U, \lambda_V, \lambda_W)}{\partial \theta_E}. \quad (5)$$

Equations (4) and (5) indicate that both the phase current and the torque can be obtained as a function of the flux linkage and the electrical angle by partial derivative of the magnetic energy. Because the flux linkage is the time integration of the voltage applied to the phase windings, the magnetic energy suffices to define the mechanical and electrical behavior of the SRM.

B. Modeling Theory

This subsection constructs the modeling theory based on the basic theory discussed in the previous subsection. Specifically, the magnetic energy is formulated as a function of the flux linkage and the electrical angle. In this paper, we neglect the magnetic coupling between the phase windings, for convenience. Because of the geometrical symmetry between the phases, we can express the magnetic energy as

$$E(\theta_E, \lambda_U, \lambda_V, \lambda_W) = E'(\theta_E, \lambda_U) + E'(\theta_E + 2\pi/3, \lambda_V) + E'(\theta_E + 4\pi/3, \lambda_W), \quad (6)$$

where E' is the magnetic energy contributed from phase U.

Magnetic energy E' has dependency on the flux linkage λ_U and the electrical angle θ_E . If we set zero of θ_E at the aligned position of phase U, we can approximate E' as

$$E'(\theta_E, \lambda_U) \approx M_1(\lambda_U) + M_2(\lambda_U)\cos\theta_E + M_3(\lambda_U)\cos 2\theta_E + \dots + M_n(\lambda_U)\cos(n-1)\theta_E, \quad (7)$$

where n is a natural number and M_1, M_2, \dots are Fourier cosine coefficients as function of λ_U .

Then, M_1, M_2, \dots are approximated using Taylor expansion as

$$M_k(\lambda_U) \approx M_{k1}\lambda_U^2 + M_{k2}\lambda_U^3 + M_{k3}\lambda_U^4 + \dots + M_{km}\lambda_U^{m+1}, \quad (8)$$

where k is an arbitrary number between 1 and n . Note that right-hand side of (8) does not have a term of the first order of λ_U because the phase current should be zero at $\lambda_U=0$.

As a result, we can model the magnetic energy in a matrix form as

$$E'(\theta_E, \lambda_U) \approx \mathbf{c}'(\theta_E)\mathbf{M}\boldsymbol{\lambda}_U, \quad (9)$$

where $\mathbf{c}'(\theta_E)$ is a vector defined as $\mathbf{c}'(\theta_E) \equiv [1, \cos\theta_E, \cos 2\theta_E, \dots, \cos(n-1)\theta_E]$, \mathbf{M} is a matrix of coefficients M_{kj} defined as $\mathbf{M} \equiv \{M_{kj}; 1 \leq k \leq n, 1 \leq j \leq m\}$, and $\boldsymbol{\lambda}_U$ is a vector defined as $\boldsymbol{\lambda}'_U \equiv [\lambda^2_U, \lambda^3_U, \lambda^4_U, \dots, \lambda^{m+1}_U]$.

Substituting (6) and (9) into (4), we can straightforwardly obtain the phase current using matrix \mathbf{M} as

$$\begin{aligned} \dot{q}_U &= \mathbf{c}'(\theta_E)\mathbf{M}\boldsymbol{\lambda}'_U, & \dot{q}_V &= \mathbf{c}'(\theta_E + 2\pi/3)\mathbf{M}\boldsymbol{\lambda}'_V, \\ \dot{q}_W &= \mathbf{c}'(\theta_E + 4\pi/3)\mathbf{M}\boldsymbol{\lambda}'_W, \end{aligned} \quad (10)$$

where $\boldsymbol{\lambda}'_U, \boldsymbol{\lambda}'_V$, and $\boldsymbol{\lambda}'_W$ are vectors defined as $\boldsymbol{\lambda}'_U \equiv [2\lambda_U, 3\lambda^2_U, 4\lambda^3_U, \dots, (m+1)\lambda^m_U]$, $\boldsymbol{\lambda}'_V \equiv [2\lambda_V, 3\lambda^2_V, 4\lambda^3_V, \dots, (m+1)\lambda^m_V]$, and $\boldsymbol{\lambda}'_W \equiv [2\lambda_W, 3\lambda^2_W, 4\lambda^3_W, \dots, (m+1)\lambda^m_W]$, respectively.

In addition, substituting (6) and (9) into (5), we can straightforwardly obtain the torque as

$$\begin{aligned} \tau &= -P\{\mathbf{c}''(\theta_E)\mathbf{M}\boldsymbol{\lambda}_U + \mathbf{c}''(\theta_E + 2\pi/3)\mathbf{M}\boldsymbol{\lambda}_V \\ &\quad + \mathbf{c}''(\theta_E + 4\pi/3)\mathbf{M}\boldsymbol{\lambda}_W\}, \end{aligned} \quad (11)$$

where $\mathbf{c}''(\theta_E)$ is a vector defined as $\mathbf{c}''(\theta_E) \equiv [0, -\sin\theta_E, -2\sin 2\theta_E, \dots, -n\sin n\theta_E]$.

As we have seen above, matrix \mathbf{M} represents the mechanical and electrical nonlinearity of the SRM. In other words, \mathbf{M} works as a simple behavior model of SRMs. In the next section, we present how matrix \mathbf{M} is determined for practical SRMs.

C. Parameter Extraction Method

Matrix \mathbf{M} can be determined experimentally by measurement of the magnetization energy of one phase, according to the following procedure.

First, E' , i.e. the magnetic energy contributed by phase U, is measured. The rotor of the motor under test is mechanically

fixed at a predetermined mechanical angle. Then, the square voltage waveform, as illustrated in Fig. 2(a), is applied as voltage V_U of phase U, whereas other phases are left open. At the same time, the phase current i_U is measured. An example of typical waveform of i_U is illustrated in Fig. 2(b). Then, we can obtain instantaneous E' and λ_U as follows:

$$E' = \int i_U(V_U - R i_U) dt, \quad (12)$$

$$\lambda_U = \int (V_U - R i_U) dt, \quad (13)$$

where R is the resistance of phase U and t is the time. Based on these experimental data, we can obtain $E'(\lambda_U)$ as a function of λ_U at the predetermined mechanical angle. Dependency of $E'(\lambda_U)$ on the mechanical angle θ_E is obtained by repeating the above procedure at various mechanical angles. Consequently, a complete database of $E'(\theta_E, \lambda_U)$ can be obtained.

Next, Fourier cosine coefficients $M_1(\lambda_U), M_2(\lambda_U), \dots$ are determined based on the database of $E'(\theta_E, \lambda_U)$. We regard the magnetic energy $E'(\theta_E, \lambda_U)$ at a predetermined flux linkage value as a function of the mechanical angle θ_E . Then, we apply Fourier expansion to the function to determine Fourier cosine coefficients. Dependency of the Fourier cosine coefficients on λ_U is obtained by repeating the above procedure at various flux linkage value.

Finally, we further apply linear regression analysis to the Fourier cosine coefficients to obtain Taylor expansion coefficients M_{kj} , which are the elements of matrix \mathbf{M} .

This parameter extraction method is similar to that proposed in [23], which is employed to determine the matrix representing dependency of the phase current on λ_U . However, in [23], Taylor expansion coefficients are first determined; and then, Fourier expansion coefficients are determined, contrarily to the proposed method.

This previous method reported in [23] can also be applied to determine matrix \mathbf{M} from the database of $E'(\theta_E, \lambda_U)$. In this

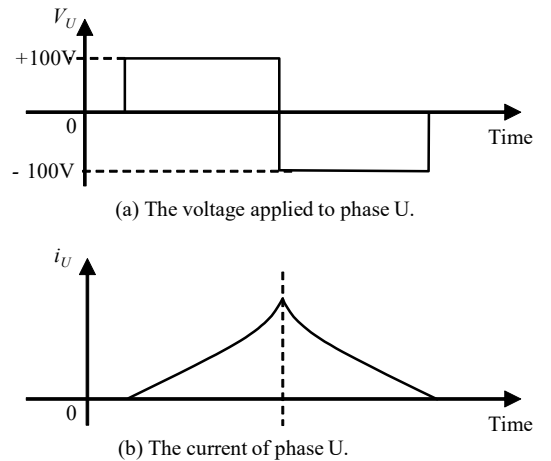


Fig. 2. Voltage and current waveforms in measurement of the magnetic energy E' contributed by phase U.

case, we first approximate $E'(\theta_E, \lambda_U)$ using Taylor expansion:

$$E'(\theta_E, \lambda_U) \approx M'_1(\theta_E) \lambda_U^2 + M'_2(\theta_E) \lambda_U^3 + M'_3(\theta_E) \lambda_U^4 + \dots + M'_m(\theta_E) \lambda_U^{m+1}, \quad (14)$$

where M'_1, M'_2, \dots are Taylor expansion coefficients as function of θ_E . Then, M'_1, M'_2, \dots are approximated using Fourier expansion as

$$M'_j(\theta_E) \approx M_{1j} + M_{2j} \cos \theta_E + M_{3j} \cos 2\theta_E + M_{4j} \cos 3\theta_E + \dots + M_{nj} \cos(n-1)\theta_E. \quad (15)$$

As a result, the matrix elements of \mathbf{M} is obtained.

As shown later in the experiment, matrix \mathbf{M} by the proposed method modeled $E'(\theta_E, \lambda_U)$ with a smaller matrix size compared with the previous method. Therefore, it appears that Fourier expansion should be first applied rather than Taylor expansion.

The reason is not cleared in this paper. However, linear regression analysis used for determining Taylor expansion coefficients is a statistical method based on optimistic assumption that the residual is always normally distributed. On the other hand, Fourier expansion does not require this assumption owing to the orthogonality of triangular functions. Therefore, Taylor expansion coefficients may be less precisely determined compared with Fourier cosine coefficients.

As a result, $M'_j(\theta_E)$ may contain extremely high harmonics caused by significant error generated during linear regression analysis. Hence, Fourier expansion of $M'_j(\theta_E)$ may require to consider high harmonics, leading to large size of matrix \mathbf{M} in the previous method. Contrarily, $M_k(\lambda_U)$ may be comparatively precisely determined in the proposed method; and therefore, $M_k(\lambda_U)$ may be well approximated in (8) with Taylor expansion with small order. Consequently, the proposed parameter extraction method might construct \mathbf{M} with a small sized matrix.

III. EXPERIMENT

Experiment is carried out to verify the proposed model. Figure 3 shows the motor test bench employed for the experiment. A SRM and a hysteresis brake is mechanically coupled together via a torque meter. Table I shows the specifications of the test bench.

A. Model Construction

First, we constructed the model for the experimental motor. We measured the magnetizing curves at 14 electrical angles in 0–180 degrees, according to the method described in Section II.C. The measurement result is shown in Fig. 4. Then, the magnetic energy was calculated based on the magnetizing curves, as shown in Fig. 5. The result was utilized to construct the database of the magnetic energy $E'(\theta_E, \lambda_U)$, i.e. a table of magnetic energy in 28 electrical angles and 100 flux linkage levels.

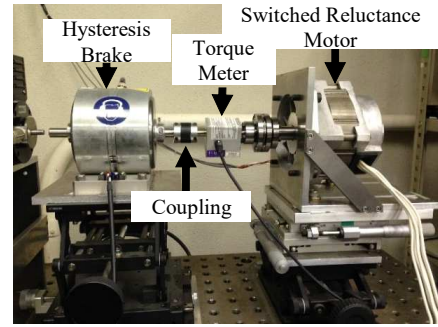


Fig. 3. Photograph of the experimental motor test bench.

TABLE I. SPECIFICATIONS OF THE MOTOR TEST BENCH

Instrument	Specifications
Motor	RB165SR-96VSRM (Motion System Tech. Inc.)
	1.2kW, 96V, 6000rpm
	Stator: 12 poles, Rotor: 8 poles.
	Number of turns: 14T
Torque Meter	UTMII-5Nm(Unipulse Corp.)
Hysteresis Brake	AHB-6 (Magtrol Inc.)

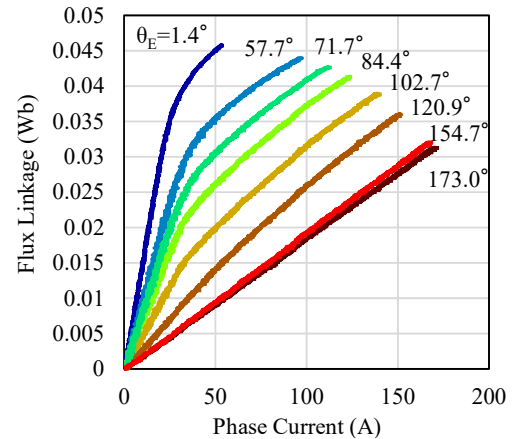


Fig. 4. Measurement results of the magnetizing curves of phase U.

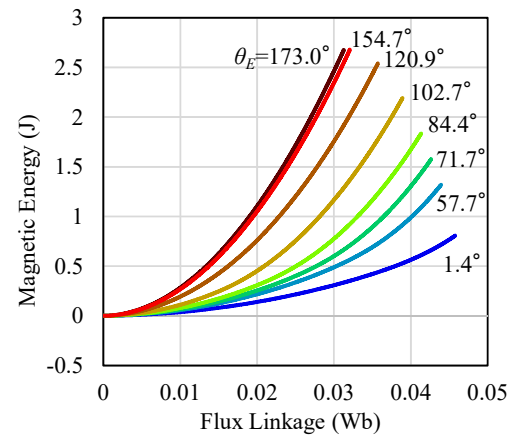


Fig. 5. Measurement results of the magnetic energy of phase U.

We modeled the magnetic energy according to the proposed parameter extraction method. We applied the Fourier expansion of order 4 to the database so that the magnetic energy is approximated with the coefficient of determination of 99.5%. Figure 6 shows the resultant Fourier cosine coefficients M_1, M_2, \dots . Then, we approximated $M_1(\lambda_U), M_2(\lambda_U), \dots$ using (8). The maximum order of λ_U is determined as 5 to achieve the coefficient of determination of 99.9%. As a result, matrix \mathbf{M} is determined as presented in Table II.

We also modeled the magnetic energy according to the previous parameter extraction method presented in [23]. We applied Taylor expansion of order 4 to the database so that the magnetic energy is approximated with the coefficient of determination of 99.9%, similarly as in the proposed parameter extraction method. Figure 7 shows the resultant Taylor

expansion coefficients $M'_1(\theta_E), M'_2(\theta_E), \dots$. Then, we approximated $M'_1(\theta_E), M'_2(\theta_E), \dots$ using Fourier expansion as in (15). The maximum order of θ_E is determined as 12 to achieve the coefficient of determination of 99.5%. As a result, matrix \mathbf{M} is determined as presented in Table III.

Comparison between Table II and Table III shows that the proposed method can model the magnetic energy with smaller size of \mathbf{M} compared with the previous method. This reduction of matrix size was mainly contributed by smooth dependency of $M_1(\lambda_U), M_2(\lambda_U), \dots$ on λ_U . On the other hand, $M'_1(\theta_E), M'_2(\theta_E), \dots$ contained high harmonics of θ_E , thus resulting in large size of \mathbf{M} . The result implies that the proposed parameter extraction method is advantageous in constructing a small-sized model.

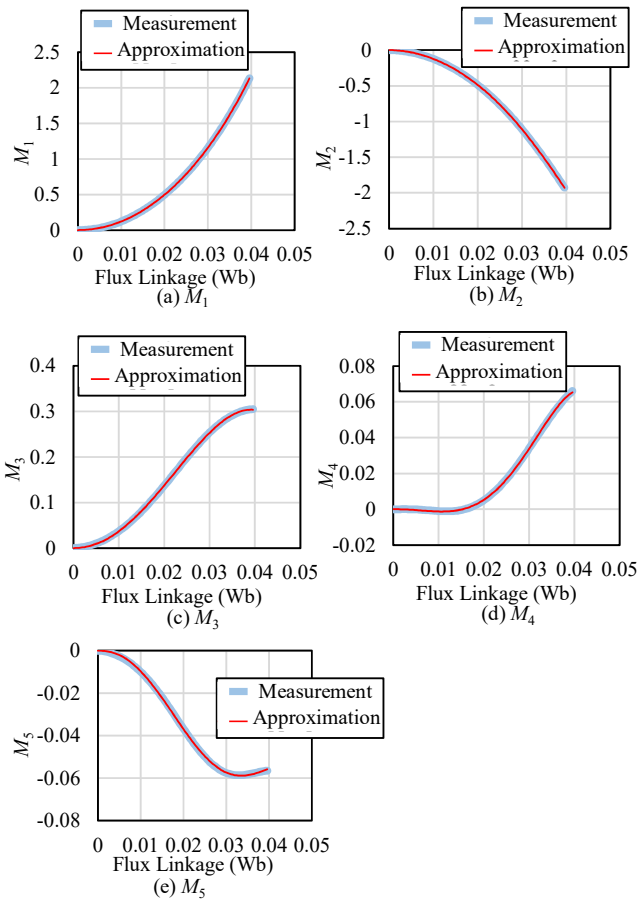


Fig. 7. Fourier cosine coefficients determined by the proposed parameter extraction method

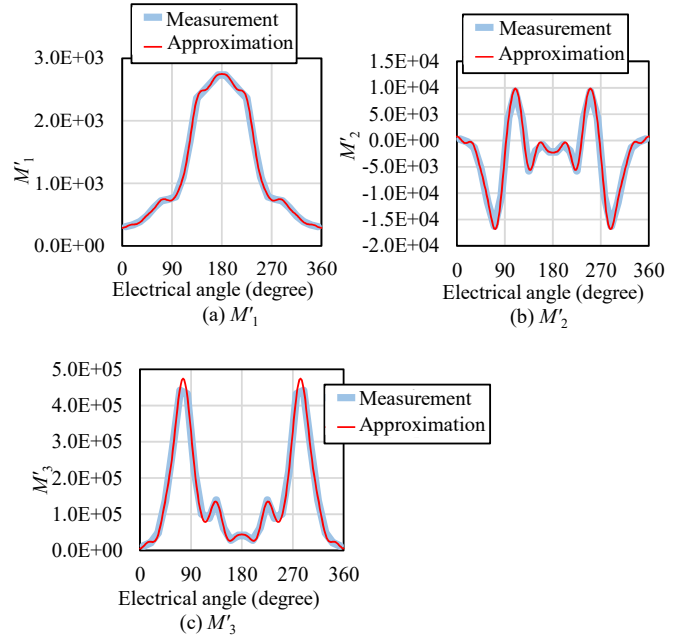


Fig. 6. Taylor expansion coefficients determined by the previous parameter extraction method

TABLE III. MATRIX \mathbf{M} DETERMINED BY THE PROPOSED PARAMETER EXTRACTION METHOD

	λ_U^2	λ_U^3	λ_U^4	λ_U^5
1	1.19.E+03	3.17.E+03	-7.59.E+04	2.66.E+06
$\cos\theta_E$	-1.35.E+03	1.70.E+04	-6.95.E+05	8.64.E+06
$\cos 2\theta_E$	3.46.E+02	4.87.E+03	-2.80.E+05	1.50.E+06
$\cos 3\theta_E$	-1.99.E+01	-8.19.E+02	1.88.E+05	-3.24.E+06
$\cos 4\theta_E$	-5.43.E+01	-7.42.E+03	3.54.E+05	-3.91.E+06

TABLE II. MATRIX \mathbf{M} DETERMINED BY THE PREVIOUS PARAMETER EXTRACTION METHOD

	λ_U^2	λ_U^3	λ_U^4
1	1.24.E+03	-2.73.E+03	1.47.E+05
$\cos\theta_E$	-1.19.E+03	-2.12.E+03	2.71.E+04
$\cos 2\theta_E$	3.74.E+02	1.53.E+03	-1.54.E+05
$\cos 3\theta_E$	-7.93.E+01	6.36.E+03	-8.24.E+04
$\cos 4\theta_E$	-1.26.E+02	1.25.E+03	2.73.E+04
$\cos 5\theta_E$	1.09.E+02	-5.36.E+03	7.21.E+04
$\cos 6\theta_E$	1.80.E+01	-4.59.E+02	6.02.E+01
$\cos 7\theta_E$	-6.02.E+01	2.64.E+03	-3.78.E+04
$\cos 8\theta_E$	3.69.E+01	-9.40.E+02	1.08.E+04
$\cos 9\theta_E$	-1.94.E+01	1.22.E+02	3.69.E+03
$\cos 10\theta_E$	1.43.E+00	-7.14.E+01	2.40.E+03
$\cos 11\theta_E$	1.26.E+01	-1.04.E+02	-2.03.E+03
$\cos 12\theta_E$	-2.20.E+01	6.43.E+02	-8.32.E+03

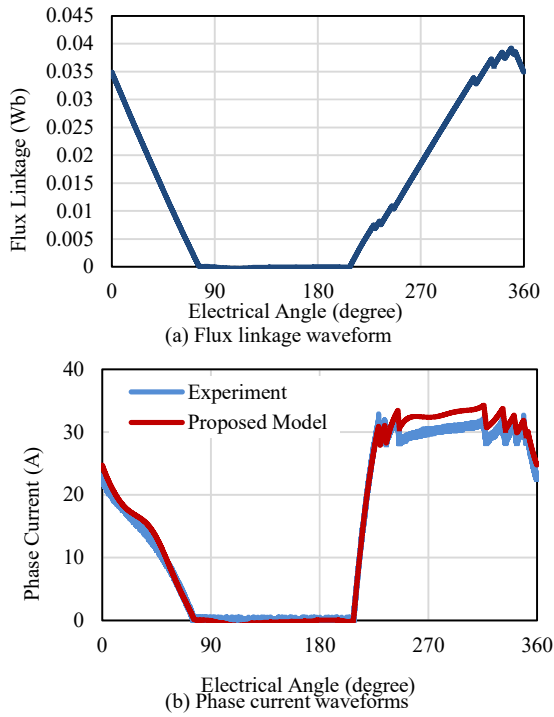


Fig. 8. Comparison of current waveforms between the experiment and the proposed model under flux linkage waveform A

B. Evaluation of the Model

Next, the proposed model shown in Table II was evaluated by predicting the phase current and the average torque of the experimental motor. For this purpose, we drove the motor with two types of the voltage waveforms. As a result, two types of flux linkage waveforms were induced as presented in Fig. 8(a) and Fig. 9(a). The rotating velocity of the motor was set at 1000rpm approximately.

The evaluation results are shown in Figs. 8–10. Figure 8(b) and Figure 9(b) present the phase current. Figure 10 presents the average torque. These evaluation results show that both the torque and the phase current waveforms are well predicted by the proposed model. The maximum deviation of the model from the experiment was found to be within 10% of the peak current in the current waveforms. In addition, the model predicted the torque within an error of 15%. These results support appropriateness of the proposed model for SRMs.

IV. CONCLUSIONS

Control techniques of SRMs is requiring simple analytical models for mechanical and electrical nonlinearity of the motor. As a promising solution, this paper presented an analytical model based on the magnetic energy. The proposed model adopts the flux linkage and the electrical angle as the state variables. In addition, the proposed model expresses the nonlinearity by a matrix that represents the magnetic energy. A practical parameter extraction method is also presented to minimize the size of the matrix.

An experiment was carried out to evaluate the parameter extraction method and the proposed model. As a result, the

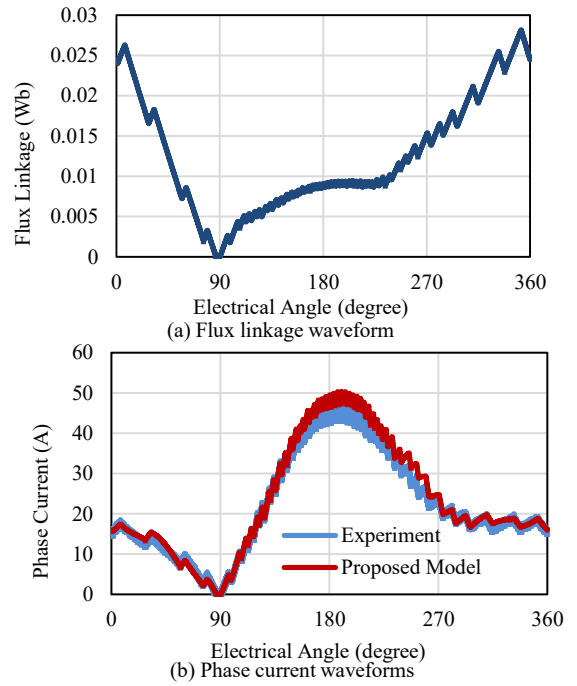


Fig. 9. Comparison of current waveforms between the experiment and the proposed model under flux linkage waveform B

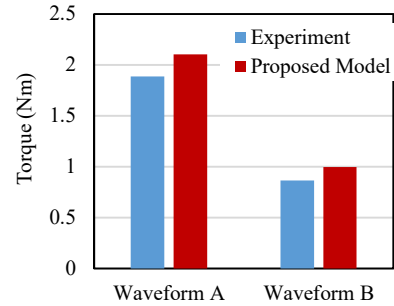


Fig. 10. Comparison of average torque between the experiment and the model under flux linkage waveforms A and B

proposed extraction method successfully derived the small-sized matrix that represents the magnetic energy. In addition, the proposed model successfully predicted the torque and phase current waveforms. Consequently, we concluded that the proposed model is promising for an analytical model for SRMs.

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